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Three Essays on Financial Macroeconomics

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Three Essays on Financial Macroeconomics

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Dissertation

Presented to the Graduate School of

the University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Doctor of Philosophy

The University of Texas at Austin

August, 2004

Three Essays on Financial Macroeconomics

Publication No. _____

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The University of Texas at Austin, 2004

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I study financial arrangements that arise in economies with limited enforcement. Contractual promises are required to be rational for the obligated party at the time of fulfillment. Common also to each environment is perfect information. I study each economy in general equilibrium with competitive markets.

In the first chapter, I study the provision of liquidity by one cohort of private agents to another building on the three-period model of Holmström and Tirole (*Journal of Political Economy*, 1998). Entrepreneurs issue financial liabilities to finance illiquid investment. As a precaution against a random cost shock, entrepreneurs optimally buy, hold, and then sell a security that they cannot issue themselves. In contrast to Holmström and Tirole, I do not allow government liabilities to serve this purpose. Instead, I require that entrepreneurs' liquidity needs be satisfied endogenously by circulation of third-party liabilities. The appropriate liabilities sell at a price premium relative to securities that do not serve the liquidity need. Liquidity

uncertainty can distort production allocations among producers with different risk characteristics, and I show how issuers of circulating liabilities may be interpreted as banks.

The second chapter presents an infinite time-horizon exchange economy wherein default cannot be punished by complete banishment from markets. An asset exists in the economy that cannot be confiscated, and that agents can never be prevented from trading. The payoff to an agent in default is a function of present and future prices and the agent's ownership share of the "non-collateral" asset. Greater ownership implies a higher payoff upon default; but a higher default payoff reduces trading opportunities in equilibrium. Equilibration may generate volatile time-series for endogenous variables. I document the quantitative implications by computing equilibria of a plausibly calibrated economy.

In the last chapter, I study the ability of a simple limited enforcement economy to explain arbitrary panel consumption data. Subject to satisfaction of mild inequality restrictions, if the consumption allocation implies that each agent's wealth is finite, there is a feasible punishment institution that induces the data in equilibrium. The result shows that limited enforcement economies hold significant potential to explain anomalous features and implications of such data.

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Chapter I

Liquidity Provision by Private Agents

1 Introduction

Economists are accustomed to thinking of the supply of money and liquidity as the province of government. While it is acknowledged in the abstract that private institutions may “create” money, the micro-foundations for this phenomenon are seldom taken very seriously, or assigned a prominent role by monetary theory. One explanation for this state of affairs is that the monetary authority is often thought to be able to affect the supply of liquidity in a manner that mimics laissez faire institutions. An alternative doctrine is that private institutions ought not to play a role in the monetary system. In fact, the operation of private liquidity provision is not well understood, and important issues that have been debated since the time of Adam Smith remain unsettled.

In this paper, I construct a model of liquidity constrained investment in which private liabilities are fully substitutable for any function that could be served by outside assets. In particular, I investigate how and to what extent the liquidity needs of agents with investment opportunities can be met endogenously in the absence of

outside assets and without enlightened intervention. The theory suggests an important link between the nature of available investment projects and the availability of liquidity in an economy, so that the liquidity supply function inherits the elasticity properties of investment. The model also admits a surprising explanation for liquidation of investment projects in an environment of liquidity crisis under perfect information.

The notion of liquidity I employ is a simple one: liquidity is the means by which wealth can be stored intertemporally. This definition is the one used by Holmström and Tirole (1998) and Kiyotaki and Moore (2001). In each of these papers (as in mine), an entrepreneur needs some form of security in order to ensure his access to capital at the right time, and his efforts are hindered by enforcement frictions in two dimensions. First, moral hazard at the time of production (in the future) generates a wedge between the rate of return on private investment and that demanded by the market. This implies that the entrepreneur cannot leverage the full value of his project, so that liquidity (as defined above) can help him capitalize on his investment opportunity. Second, futures markets do not operate efficiently, so that liquidity may be scarce.

Kiyotaki and Moore (2001) show that a fungible asset in fixed supply can earn an extra-fundamental “liquidity premium” (that is, a price higher than that consistent

with its marginal product) and circulate like money. In their model, land serves a collateral function in addition to its role as an input to production. Here the scarcity of land is an exogenous property, and its price is determined endogenously. In Holmström and Tirole, the liquidity need is met by a government asset that is supplied perfectly elastically at a price fixed exogenously. They show that the asset may be demanded even if the price is higher than the fundamental one.

I adopt salient features of the model of Holmström and Tirole into my model, but I depart from them by requiring that liquidity be supplied endogenously: there is no government in my model. Instead, a second producer (whom I call the “banker”) is introduced who is capable of issuing securities, backed by his project dividends, that can be used by the entrepreneur to mitigate his liquidity needs. Notice that the liquid securities in my economy are backed by investment, and to the extent that the banker’s investment depends on the market rate of interest, his issue of liquid liabilities will do so as well. This phenomenon induces an upward-sloping liquidity supply curve, which contrasts the vertical one in Kiyotaki and Moore (2001) and the horizontal one in Holmström and Tirole.

Somewhat surprisingly, it is possible for liquidation of the banker’s assets to occur in equilibrium. This is because I will assume that the rate of return wedge described above affects also the banker, except when the banker terminates his project early.

That is, the moral hazard problem associated with management of the banker's project may be circumvented by liquidation. Therefore, even though the net return on liquidated investment may be low, it may be that the banker can promise to pay more to outside stakeholders when his project is liquidated than when it is allowed to mature. This property engenders an interesting gambling behavior wherein the banker terminates his investment in a state in which the liquidity need of the entrepreneur is particularly acute.

The mechanism for liquidity provision in the model brings to mind the “real bills doctrine” that private financial instruments backed by appropriate assets should be allowed to supplement other media of exchange. Sargent and Wallace (1982) and Champ, Smith and Williamson (1996) each construct models in which institutions that afford money creation under laissez faire conditions impart a beneficial elasticity to the supply of money. There is no scarcity of money per se, because these models abstract from commitment frictions that might disqualify certain assets from serving as security for a note. In Sargent and Wallace, for example, each intertemporal trading opportunity may be assumed to give rise to a risk-free bill of exchange for the full value of the desired transaction. Allocations are never constrained by the quantity of money that can be conjured, because the issue of a note is assumed to induce an obligation that does not admit default. The model of the present paper

contributes to the understanding of the elasticity of the transactions medium by showing how, when contracting frictions exist, valuable trading opportunities may be missed due to a shortage of liabilities suitable for circulation. Whereas money creation and credit creation are identical in these other environments, my model implies an endogenous separation between the assets and the liabilities of the banker even in a *laissez faire* setting.

With respect to the interpretation of institutions in the model, this paper is similar to Gorton and Pennacchi (1990). These authors investigate the transactions value of riskless securities for Diamond-Dybvig-style liquidity traders, and rationalize the circulation of those securities in equilibrium. The instruments used in these transactions are interpreted alternatively as corporate debt or bank deposits, each serving the same function in the model. Comparing the transactions velocities of bank deposits and corporate debt, the authors find unsupported the empirical conclusion that corporate debt serves the role ascribed in the model. However, it may be more appropriate to make the comparison to savings deposits or CDs, as these seem more attuned to meeting liquidity needs occurring at business cycle frequencies. I explore this argument in the discussion section of the paper.

The position that the supply and availability of private liabilities affects the allocation of real resources can be supported empirically. First, while causality from

money to output is notoriously difficult to establish, it is known that broad measures of money incorporating interest-bearing private marketable liabilities are more highly correlated with output than the monetary base or M1.¹ Thus were Friedman and Schwartz (1963) motivated to focus on M2 in their famous study. Secondly, Friedman and Kuttner (1993) have found that the six-month commercial paper rate is more informative with respect to movements of output than the rate of interest on the three-month T-bill. Other researchers have verified that this finding is quite robust. At a minimum, this evidence suggests that the liabilities of the government do not determine the financial environment independent of the positions and capabilities of other participants.

The rest of the paper is structured as follows. In the next section, I introduce the formal model and assumptions. In the third section, I conduct the formal analysis of the model. The fourth section discusses my findings and conjectures implications that might be derived from various extensions. The last section summarizes and concludes.

¹See, for example, Cooley and Hansen (1995).

2 Environment

2.1 Time, Preferences, and Endowments

The economy is inhabited by three agents, whom I label as the banker, the entrepreneur, and the worker. I index these agents by $i \in \{b, e, w\}$. There are three dates $t \in \{0, 1, 2\}$.

There is a single good in each period in the economy. The good is useful for consumption at any date, and the dates 0 and 1 goods are useful also for investment in production projects as discussed below. The date t good perishes if it is not consumed or invested by the end of that date. At date 0, the banker and the entrepreneur have endowments ω^b and ω^e , respectively. The worker has no endowment of the good at date 0, and no agent is endowed with goods at dates 1 or 2.

The worker has an unlimited quantity of labor at dates 0 and 1 that may be converted one-for-one into the contemporaneous good. Producing goods in this manner has a disutility for the worker equivalent to one unit of consumption. The worker's labor is inalienable, so that a promise from the worker to provide labor in the future can be reneged with impunity.²

The banker and the entrepreneur each have a production project that can be

²I provide a precise mathematical representation for this concept below.

used to produce date 2 goods subject to a pattern of investment of goods at date 0 and 1. Projects are discussed in detail in the next subsection.

All agents are risk neutral. The banker and the entrepreneur evaluate outcomes according to the sum of non-negative consumption at the three dates. The worker evaluates outcomes according to the sum of consumption at each date minus labor expended. At each date, agents act in order to maximize the expectation of their payoff at the current and future date(s). Agents do not discount the future.

2.2 Production Projects

Production in the economy is affected by the realization of a random variable that is observed at date 1. The random variable takes on the value H with probability ϕ , and L with probability $1 - \phi$. For convenience of notation, I will sometimes write $s \in S = \{H, L\}$ for a generic outcome, and I write ϕ_s for the probability that the outcome is s . I will refer to a pairing of the date and the realization as the state; that is, state 1s indicates date 1 when the realized outcome outcome is s . Abusing the notation, I may refer to the state at date 0 as "state 0".

I first describe the project of the entrepreneur.

At date 0, the entrepreneur chooses investment level $I^e \in \mathbb{R}_+$. At date 1, the project may need additional investment of goods in order to continue. Precisely, if

the outcome from the random variable at date 1 is H , then the project requires additional investment of $(1 - \lambda_H^e) I^e \rho$, where $\lambda_s^e \in [0, 1]$ is the fraction of his project that is discontinued in state $1s$, and $\rho > 0$.³ For simplicity, I assume that no additional investment is required in state L . Discontinuance or "liquidation" of the entrepreneur's project yields no residual, so that the fraction that is liquidated is simply lost. At the beginning of date 2, the entrepreneur has an opportunity to abscond with a booty of $(1 - \lambda_s^e) I^e \gamma^e$ from the project and consume it, in which case he leaves the remnants of the project valueless. If he chooses rather to allow the project to mature, then it yields an amount $(1 - \lambda_s^e) I^e R^e$ to be divided according to agreements reached by the entrepreneur with other agents. Note that the other agents have no recourse against the entrepreneur when he absconds, but claims against the yield of a project are enforceable once the entrepreneur has chosen to allow it to mature.

Assumption 1e. I assume that the mature yield of the entrepreneur's project is greater than the value to the entrepreneur when he absconds, $R^e > \gamma^e$; and that the net surplus available from investment is positive in expectation,

$$R^e - \phi \rho - 1 > 0.$$

³In order to establish a more generalized and convenient notation, I will sometimes write $\rho_L := 0$ and $\rho_H := \rho$.

The project available to the banker is similar to that of the farmer, but never requires additional investment at date 1. This project works as follows.

At date 0, the banker chooses an investment level $I^b \in \mathbb{R}_+$. I allow for liquidation of a fraction $\lambda_s^b \in [0, 1]$ of the banker's project in state 1s. Different from the entrepreneur's project, there is a positive yield $\lambda_s^b I^b L$ available from the liquidated portion of the banker's project in state 1s, where $L < 1$. Like the entrepreneur, the banker can abscond with some amount $(1 - \lambda_s^b) I^b \gamma^b$ at the beginning of date 2, scuttling the remainder of the project without recourse by other agents. If he chooses instead to allow the project to mature, then the project yields a dividend $(1 - \lambda_s^b) I^b R^b$ to be divided according to any agreement arranged at previous dates.

Assumption 1b. The mature yield of the banker's project is greater than the value to the banker when he absconds, $R^b > \gamma^b$; and the net surplus available from investment in the banker's project is positive,

$$R^b - 1 > 0.$$

2.3 Market Institutions

I assume that, at each date $t \in \{0, 1\}$, a competitive market exists for state-contingent claims to goods at date $t + 1$. I write q_{1s} for the date 0 price of claims to

goods in state $1s$, and I write q_{2s} for the state $1s$ price of claims to goods in state $2s$. I denote the consumption of agent i in state σ by c_σ^i , I denote the labor supplied by the worker in state σ by n_σ , and I denote the net claims to state ts consumption held by agent i at the end of date $t - 1$ by B_{ts}^i .

2.3.1 The Worker's Problem

The worker takes present and future claims prices as given, and chooses net claims holdings, and non-negative consumption and labor to maximize his expected lifetime consumption. The worker's choices are subject to a sequence of budget constraints, as well as time-consistency constraints reflecting the inalienability of his labor. Formulated mathematically, the worker chooses (c^w, n^w, B^w) to maximize

$$c_0^w - n_0 + \sum_{s \in S} \phi_s (c_{1s}^w - n_{1s} + c_{2s}^w). \quad (1)$$

subject to the budget constraints

$$c_0^w - n_0 + \sum_{s \in S} q_{1s} B_{1s}^w \leq 0 \quad (2)$$

$$c_{1s}^w - n_{1s} + q_{2s} B_{2s}^w \leq B_{1s}^w \text{ for each } s \in S \quad (3)$$

$$c_{2s}^w \leq B_{2s}^w \text{ for each } s \in S, \quad (4)$$

and the individual rationality constraints

$$c_{1s}^w - n_{1s} + c_{2s}^w \geq 0 \text{ for each } s \in S. \quad (5)$$

The constraints (2) and (3) require that the net purchases of the worker in the dates 0 and 1 financial markets are no greater than his wealth in the appropriate state. The wealth of the worker at date 0 is zero, and he affords financial asset purchases only by working more than he consumes. Similarly, his wealth in state $1s$ is given by his accumulation of assets from the previous period B_{1s}^w . The date 2 budget constraints (4) show that the worker consumes no more in state $2s$ than afforded by his accumulation of claims B_{2s}^w . The last set of constraints (5) reflect the fact that the worker is free to renege on any agreement to provide labor that does not benefit him ex post; this is the mathematical manifestation of inalienable labor.

Inspecting this problem, it is obvious that any solution admits a continuum of alternative solutions in which the worker increases his labor effort and consumption in some state by the same amount. Therefore, I will avoid ambiguities and simplify the problem somewhat by characterizing the worker's choice of net labor supply $x_0 := n_0 - c_0^w$ and $x_{1s} := n_{1s} - c_{1s}^w$ at dates 0 and 1, rather than specifying precisely his consumption and labor supply at these dates. The problem of the worker under this transformation can be stated in the obvious way.

2.3.2 The Entrepreneur's Problem

The entrepreneur takes current and future claims prices as given and chooses net claims holdings, non-negative consumption and investment, and a project liquidation rule $(c^e, I^e, \lambda^e, B^e)$ to maximize his expected lifetime payoff. The objective function of the entrepreneur is

$$c_0^e + \sum_s \phi_s (c_{1s}^e + c_{2s}^e), \quad (6)$$

and his budget constraints are

$$c_0^e + I^e + \sum_{s \in S} q_{1s} B_{1s}^e \leq \omega^e \quad (7)$$

$$c_{1s}^e + I^e (1 - \lambda_s^e) \rho_s + q_{2s} B_{2s}^e \leq B_{1s}^e \text{ for each } s \in S \quad (8)$$

$$c_{2s}^e \leq B_{2s}^e + I^e (1 - \lambda_s^e) R^e \text{ for each } s \in S. \quad (9)$$

The date 0 budget constraint (7) says that the entrepreneur can apply no more than his endowment to date 0 consumption, investment, and purchase of financial claims. The date 1 budget constraints (8) say that accumulated claims must be used to fund consumption, additional investment, and the portfolio to be held at date 2. Finally, (9) says that date 2 consumption must be funded out of accumulated claims and project dividends. Clearly, the last budget constraint is valid for the case that the entrepreneur does not abscond with the project dividends; I discuss this possibility

next.

Since the entrepreneur can always achieve consumption of $(1 - \lambda_s^e) I^e \gamma^e$ at date 2 by his choice to abscond, allocations face an additional constraint in this regard. To see this, suppose that the entrepreneur holds claims $B_{2s}^e < -I^e (1 - \lambda_s^e) (R^e - \gamma^e) < 0$ for some state at date 2. Then (9) implies that $c_{2s}^e < (1 - \lambda_s^e) I^e \gamma^e$, and the entrepreneur can achieve a higher payoff by absconding. Under perfect information, no agent would buy a quantity of claims from the entrepreneur that would induce him to abscond at date 2. Equivalently, it must be that allocations for the entrepreneur satisfy the *incentive compatibility constraints*

$$c_{2s}^e \geq (1 - \lambda_s^e) I^e \gamma^e \text{ for each } s \in S. \quad (10)$$

The problem of the entrepreneur may now be stated as that of maximizing (6) subject to (7)-(10).

The object $\tilde{R}^e := R^e - \gamma^e$, which is positive by Assumption 1e, plays an important role in the analysis to follow. The simple moral hazard problem described above creates a wedge between the internal rate of return available to the entrepreneur through investment, and the share that can be pledged to outsiders. \tilde{R}^e is the *marketable share* of the entrepreneur's project, the maximal amount that can credibly be pledged to outside stake-holders per unit of investment.

Assumption 2e. The marketable share of the entrepreneur's project satisfies, for all liquidation rules $\lambda^e \in [0, 1]$,

$$(1 - \phi) \lambda_L^e \tilde{R}^e + \phi \lambda_H^e (\tilde{R}^e - \rho) < 1.$$

This assumption can be seen to imply that, for sufficiently large investment I^e , the entrepreneur will be unable to credibly promise to repay $I^e - \omega^e$ to outsiders; in particular, the entrepreneur will be unable to finance an arbitrarily high amount of investment.

2.3.3 The Banker's Problem

The problem of the banker is very similar to that of the entrepreneur. The banker takes prices as given and chooses net claims holdings, non-negative consumption and investment, and a project liquidation rule $(c^b, I^b, \lambda^b, B^b)$ to maximize the objective function

$$c_0^b + \sum_{s \in S} \phi_s (c_{1s}^b + c_{2s}^b) \tag{11}$$

subject to the budget constraints

$$c_0^b + I^b + \sum_{s \in S} q_{1s} B_{1s}^b \leq \omega^b \quad (12)$$

$$c_{1s}^b + q_{2s} B_{2s}^b \leq B_{1s}^b + I^b \lambda_s^b L \text{ for each } s \in S \quad (13)$$

$$c_{2s}^b \leq B_{2s}^b + I^b (1 - \lambda_s^b) R^b \text{ for each } s \in S, \quad (14)$$

and the incentive compatibility constraints

$$c_{2s}^b \geq I^b (1 - \lambda_s^b) \gamma^b \text{ for each } s \in S. \quad (15)$$

The constraints (12)-(14) are to be interpreted analogously to (7)-(9) for the entrepreneur, with the principle exception that the banker has no need of additional funding at date 1, and instead may obtain goods at that date by liquidating a portion of his project. The incentive compatibility constraint (15) is derived and interpreted analogously to (10).

Like that of the entrepreneur, the *marketable share* of the banker's project, defined as $\tilde{R}^b := R^b - \gamma^b$, plays an important role in the analysis to follow. I make the following assumption analogous to Assumption 2e.

Assumption 2b. The marketable share of the banker's project is less than one, $\tilde{R}^b < 1$.

2.3.4 Definition of Equilibrium

In the analysis to follow, the central objects of interest will be the issue and purchase of claims by agents in the model. I follow this route because the interpretations I will offer focus on properties of financial markets in equilibrium. Given this agenda, it is sensible to posit a definition of equilibrium in terms of these markets, rather than the market for goods. The following definition is, of course, equivalent to one that incorporates a goods market clearing condition

An *equilibrium* is an allocation $\{c, n, I, \lambda, B, q\}$ of consumption, labor, investment, liquidation policies, net assets holdings, and asset prices such that (i) each agent's problem is solved; and (ii) the markets for contingent claims to consumption clear at dates 0 and 1, i.e., for each $t \in \{1, 2\}$ and $s \in \{H, L\}$, $\sum_i B_{ts}^i = 0$.

It is easy to see that each agent's marginal rate of substitution of consumption at date 0 for consumption in state 1s is ϕ_s , and that of consumption in state 1s for consumption in state 2s is 1. Therefore, I will refer to the price system defined by $q_{1s} = \phi_s$ and $q_{2s} = 1$ for each s as the *fundamental* one. As will be seen in the next section, this price system need not support an equilibrium.

The following assumption implies that the liquidity shock to the entrepreneur's project may induce a liquidity problem in the sense that the entrepreneur will need to hoard assets in order to continue his project in the bad state.

Assumption 3. The marketable share of the entrepreneur is less than the additional investment required in state 1H, $\tilde{R}^e - \rho < 0$.

3 Analysis

3.1 The Role of the Worker

Looking at the problem solved by the worker, it is clear that his optimal policy must have each budget constraint binding. By solving the dates 1 and 2 constraints and eliminating the worker's claims holdings, the problem can be restated as that of maximizing

$$-x_0 + \sum_{s \in S} \phi_s (-x_{1s} + c_{2s}^w)$$

subject to the lifetime budget constraint

$$-x_0 + \sum_{s \in S} q_{1s} (-x_{1s} + q_{2s} c_{2s}^w) \leq 0$$

and the individual rationality constraints

$$-x_{1s} + c_{2s}^w \geq 0 \text{ for each } s \in S. \tag{16}$$

(Recall that x_0 and x_{1s} are the net labor supplies of the worker in states 0 and $1s$, respectively.) Taking the first-order conditions for x_0 , x_{1s} , and c_{2s}^w , one can derive the necessary conditions, for each s ,

$$\eta_s^w = q_{1s} - \phi_s$$

and

$$1 - q_{2s} \leq 0, \quad \text{with equality if } c_{2s} > 0, \quad (17)$$

where η_s^w is a non-negative Lagrange multiplier on the worker's state $1s$ individual rationality constraint. It is immediate that the worker's problem has a solution only if $q_{1s} \geq \phi_s$ and $q_{2s} \geq 1$. These properties are a manifestation of the worker's infinite capacity to purchase assets that yield a positive return. For example, if it were true that $q_{1s} < \phi_s$, then the worker could profit by converting an arbitrarily large amount of date 0 labor into date 0 goods, and trading the date 0 goods for claims to state $1s$ consumption. Since none of his constraints are violated by making x_0 arbitrarily positive while setting $x_{1s} = -x_0/q_{1s}$, and since the net contribution of the scheme to his lifetime expected payoff is $\phi_{1s}/q_{1s} - 1 > 0$ per unit of net labor expended, the worker's problem has no solution. Since the definition of equilibrium requires that the worker's problem have a solution, such prices are precluded. Thus, I have proved

the following lemma.

Lemma 1 *If q is an equilibrium price system, then $q_{1s} \geq \phi_s$ and $q_{2s} \geq 1$ for each s .*

On the other hand, there is no symmetric argument available that shows that q_{1s} cannot be higher than ϕ_s in equilibrium. To see why, use (17) to derive the complementary slackness condition $q_{2s}c_{2s}^w = c_{2s}^w$, and note that this implies (from (3) and (4)) that

$$B_{1s}^w = -x_{1s} + c_{2s}^w.$$

Now the individual rationality constraints (16) imply that $B_{1s}^w \geq 0$ for each s . Whereas the worker can *buy* assets to take advantage of a *low* price of date 1 consumption, his inability to commit to supply future labor prohibits his *issuing* assets to take advantage of *high* prices. The following proposition is an important extension of this discussion.

Proposition 2 *If q is an equilibrium price system,⁴ there is an optimal policy for the worker with claims holdings B^w if and only if the following hold for each s :*

- (i) $B_{1s}^w \geq 0$ with equality if $q_{1s} > \phi_s$; and
- (ii) $B_{2s}^w \geq 0$ with equality if $q_{2s} > 1$.

Proof. The result is obvious from the discussion above. ■

⁴In particular, the lemma is true if $q_{1s} \geq \phi_s$ and $q_{2s} \geq 1$.

One implication of this result is that the market interest rates may not be too high, but may be low. For example, the analysis above indicates that the interest factor earned by non-contingent two-period loans, which is $R := (\sum_s q_{1s} q_{2s})^{-1}$, is no greater than each agent's marginal rate of intertemporal substitution of date 0 goods for sure claims to goods at date 2, $R \leq 1$. Moreover, it will be seen in what follows that equilibrium may have $R < 1$.

3.2 A Simple Special Case

In this subsection, I analyze the special case of the model in which the probability that the additional investment by the entrepreneur will be necessary is one; that is $\phi = 1$. This case simplifies several aspects of the exposition by allowing me to more clearly focus on the date 0 market for state 1 H claims. As will be seen in the next subsection, this market will retain its conceptual centrality in the general case even while other elements may create distraction.

The simplifications afforded in the present case are as follows. First, the dimension of the space in which claims prices lie is halved. Second, it is obvious that no agent will choose to liquidate a project, since (under perfect foresight) he could always do better by simply reducing the scale of investment ex ante. Whereas the increase of the dimension of the price space in the sequel is simply a (notational)

nuisance, the second factor represents a conceptual limitation of the present case.

The simplest case does have one unique conceptual virtue: it dispels any tendency to think about the liquidity problem studied here as inseparable from "risk". More precisely, one should take away the understanding that the liquidity need of the entrepreneur is not induced by uncertainty.

I take advantage of these simplifications by imposing that $\lambda_H^b = \lambda_H^e = 0$ a priori, and by ignoring reference to the outcome of the random process where there can be no confusion. For example, I will write q_1 rather than q_{1H} , and I will write c_1^b instead of c_{1H}^b . I do this in the present subsection only.

From the analysis of the previous subsection, it can be seen that the market return on two-period saving is less than the return available to the banker on funds invested internally. That is, $R \leq 1 < R^b$, where $R = (q_1 q_2)^{-1}$ in the present case, and the second inequality follows from Assumption 1b. As will be seen, this "wedge" can exist in equilibrium, because of the moral hazard frictions that prevent agents from exploiting the full potential of investment projects using financial markets. In this environment without a consumption smoothing motive, the banker will find it advantageous to borrow as much as possible against project proceeds and apply all of his lifetime wealth toward investment.

To see this, suppose that (c^b, I^b, B^b) is a feasible policy for the banker, and that

$c_0^b = \Delta > 0$. Now construct the alternative policy $(\tilde{c}^b, \tilde{I}^b, B^b)$ as follows. Let $\tilde{c}_0^b = 0$, $\tilde{I}^b = I^b + \Delta$, and $\tilde{c}_2^b = c_2^b + \Delta R^b$. Let the remaining elements of the new policy be identical to the old. Now it is easy to check that the new policy is feasible if the old one is. Moreover, subtracting the payoff under the old policy from that generated by the new gives $\Delta (R^b - 1) > 0$. Similarly, if one had either $c_1^b > 0$ or $c_2^b > I^b \gamma^b$, an improvement can easily be constructed by applying the date 0 financial value of the surplus to investment, and using the increase in output to augment date 2 consumption.⁵ These results are stated formally and proved for the general case in Lemma 5 below.

At an optimum, each agent's budget constraints bind, and these can be solved for the optimal investment of the banker,

$$I^b = \frac{\omega}{1 - q_1 q_2 \tilde{R}^b},$$

where one should recall the definition of the marketable share of the banker's project $\tilde{R}^b := R^b - \gamma^b$. Of particular interest for the equilibrium concept described above is the pattern of claims holdings adopted by the banker under equilibrium prices. From the period budget constraints, one has the following result.

⁵More precisely, if one had $c_1^b = \Delta$, an improving policy has $\tilde{c}_1^b = 0$ with $\tilde{I}^b = I^b + q_1 \Delta$ and $\tilde{c}_2^b = c_2^b + q_1 \Delta R^b$; and a policy with $c_2^b = I^b \gamma^b + \Delta$ admits an improvement by $\tilde{I}^b = I^b + q_1 q_2 \Delta$ and $\tilde{c}_2^b = c_2^b + \Delta (q_1 q_2 R^b - 1)$.

Proposition 3 *Suppose that additional investment by the entrepreneur is required with probability one. At equilibrium prices q , there is an optimal policy for the banker with claims holdings B^b if and only if*

$$B_1^b = -q_2 I^b \tilde{R}^b = -\frac{q_2 \omega^b \tilde{R}^b}{1 - q_1 q_2 \tilde{R}^b} \quad (18)$$

and

$$B_2^b = -I^b \tilde{R}^b = -\frac{\omega^b \tilde{R}^b}{1 - q_1 q_2 \tilde{R}^b}. \quad (19)$$

One subtlety of the presentation of this result is worth noting, as it pertains to language that I will use several times below. The hypothesis of "equilibrium prices" implies that the denominator in the expressions above must be positive. To see this, it suffices to notice that the banker could finance unlimited consumption if $1 - q_1 q_2 \tilde{R}^b$ were negative. By eliminating the claims holdings from the binding budget constraints, one can solve for the lifetime budget constraint

$$c_0^b + I^b + q_1 [c_1^b + q_2 (c_2^b - I^b \tilde{R}^b)] \leq \omega^b.$$

Then by setting $c_0^b = c_1^b = 0$ and $c_2^b = I^b \gamma^b$, it may be seen that the left-hand-side is decreasing in I^b if $1 - q_1 q_2 \tilde{R}^b \leq 0$. Since this policy satisfies the incentive constraint by construction, the banker's problem has no finite solution, contradicting the definition

of equilibrium.

This property of equilibrium has an intuitive interpretation that may be facilitated by defining the expression $\tilde{R}^b \left(1 - \tilde{R}^b/R\right)^{-1}$ as the *leverage ratio* of the banker. This is the factor by which the market prices allow the banker to multiply his own endowment to determine his maximal issue of liabilities at date 2. In equilibrium, the market interest factor R is determined by the interplay of the supply and demand for liquid claims. The negativity of the banker's claims holdings clearly shows that he acts on the supply side, as a borrower. As the market interest rate, the rate demanded by lenders, is reduced, the banker is able to issue more and more liabilities, and this amount tends to infinity as the rate approaches the marketable share of the banker. Thus, any finite demand for liquid liabilities can be quenched by the supply of the banker for some $R > \tilde{R}^b$, so that the leverage ratio will always be finite (and positive).

The essential distinction between the problem of the entrepreneur and that of the banker is that the entrepreneur must cope with the need of additional investment at date 1. Because the marketable share of the entrepreneur's project is assumed (Assumption 3) to be less than the investment need at date 1 (with probability one in the environment under consideration), it will be seen that the entrepreneur will be a net buyer of date 1 claims at date 0. This is in stark contrast to the role played

by the banker. As has been shown above, the equilibrium price of date 1 claims may be high. Therefore, the need of liquidity may impinge upon the profitability of the entrepreneur's project in a non-fundamental way. For example, it will be seen that q_1 may be high enough that the net private return from investment is not positive under any policy for the entrepreneur.

If the fundamental prices hold, then the entrepreneur will desire to invest as much in his project as possible. This case leads him to behave as the banker does, consuming nothing at dates 0 and 1, and taking the maximal amount of leverage against project proceeds by setting date 2 consumption so that his incentive compatibility constraint binds. On the other hand, if the price of future claims rises above fundamentals, then the entrepreneur will desire to transfer as much of his future consumption to date 0 as possible for any given level of investment. The principal difference between the two cases is that it may be optimal for the entrepreneur to consume his endowment (rather than invest) at date 0 if the price of liquidity is sufficiently high.

The arguments of the previous paragraph imply that $c_1^e = 0$ and $c_2^e = I^e \gamma^e$, and now the dates 1 and 2 budget constraints can be used to write optimal claims

holdings of the entrepreneur as

$$B_1^e = I^e \left(\rho - q_2 \tilde{R}^e \right) \quad (20)$$

and

$$B_2^e = -I^e \tilde{R}^e. \quad (21)$$

(Recall that $\tilde{R}^e := R^e - \gamma^e$ is the marketable share of the entrepreneur's project.)

Now it can be seen that (19) and (21) imply that $q_2 = 1$ in equilibrium; for if not, then Proposition 2 implies that $B_2^w = 0$, and $\sum_i B_2^i < 0$ contradicts the market clearing condition. In meeting the liquidity need, the entrepreneur sells at date 1 the securities he purchases at date 0 for capital goods. Thereafter, the worker holds all of the outstanding liabilities of the other two agents, a position he will only accept if the price of claims is consistent with fundamentals.

On the other hand, because the entrepreneur is a net buyer of claims to date 1 goods (that is, $B_1^e \geq 0$), the market for such claims may not clear at the fundamental price. This can be seen more easily by looking at a reduced form of the entrepreneur's problem. Solving the entrepreneur's budget constraints for date 0 consumption by eliminating his claims holdings and substituting the result into his objective function,

the problem becomes that of choosing I^e to maximize

$$I^e \left[\gamma^e - q_1 \left(\rho - \tilde{R}^e \right) - 1 \right] + \omega^e$$

subject to

$$I^e \leq \frac{\omega^e}{1 + q_1 \left(\rho - \tilde{R}^e \right)},$$

where the constraint inheres from the non-negativity of date 0 consumption. If the expression in square brackets in the objective function is positive, then each unit of investment increases the payoff of the entrepreneur, and he will invest the maximal amount allowed by the constraint. But this expression is non-positive for $q_1 \geq \hat{q}$, where

$$\hat{q} := \frac{\gamma^e - 1}{\rho - \tilde{R}^e} > 1,$$

and the inequality follows from Assumption 1e after some algebra. The next proposition is the important corollary of this discussion.

Proposition 4 *Suppose that additional investment by the entrepreneur is required with probability one. At equilibrium prices q , there is an optimal policy for the en-*

trepreneur with claims holdings B^e if and only if

$$B_1^e = \frac{\omega^e (\rho - \tilde{R}^e)}{1 + q_1 (\rho - \tilde{R}^e)} \chi$$

and

$$B_2^e = -\frac{\omega^e \tilde{R}^e}{1 + q_1 (\rho - \tilde{R}^e)} \chi$$

for some $\chi \in \Xi(q_1)$, where

$$\Xi(q_1) := \begin{cases} 1, & \text{for } q_1 < \hat{q} \\ [0, 1], & \text{for } q_1 = \hat{q} \\ 0, & \text{for } q_1 > \hat{q}. \end{cases}$$

Proof. The result is obvious from the discussion above. ■

Figure 1 shows the date 0 "supply of liquidity" by the banker and the "demand for liquidity" by the entrepreneur as functions of the price of liquidity, where these objects are defined as follows.⁶ The supply of liquidity is the amount of date 1 claims issued by the banker at date 0, which is the opposite of his claims holdings. From Proposition 3 and the fact that $q_2 = 1$ in equilibrium, the supply of liquidity is given

⁶Here, I hold the price of claims at date 2 at the equilibrium value $q_2 = 1$.

by

$$\frac{\omega^b \tilde{R}^b}{1 - q_1 \tilde{R}^b}.$$

The family of broken curves in the figure represent the supply of liquidity for different values of the banker's date 0 endowment ω^b . The demand for liquidity is the entrepreneur's holding of date 1 claims given in Proposition 3. The demand is represented by the solid curve in the figure.⁷

The supply and demand of liquidity represent a convenient device for characterizing the equilibrium price of liquidity q_1 as follows. First, if the curves do not intersect for some $q_1 \geq \phi (= 1)$, then the supply of liquidity by the banker at the fundamental price exceeds the demand of the entrepreneur; this is the case for the highest broken curve representing the largest value of ω^b in the figure. From Proposition 2, it is clear that the worker will be able and willing to purchase excess liquid claims supplied by the banker at the fundamental price $q_1 = 1$; apparently, this is the equilibrium price in this case.

Next, if the curves intersect for some price between the fundamental price and \hat{q} , as they do for the intermediate of the supply curves in the figure, then the equilibrium exhibits a liquidity price premium on liquid claims and no liquidation is required by the entrepreneur. The price premium induces the banker to borrow more than

⁷The parameters used in the plot are those of the Example 1 below.

he would under fundamental prices, essentially subsidizing his investment. At the same time, the entrepreneur reduces investment relative to what he would choose otherwise, because the added price of providing for the continuation of his project impinges on his ability to raise capital and invest. In this case, the price premium dissuades the worker from purchasing any of these claims, and all of the claims issued by the banker are purchased by the entrepreneur.

The third possibility is that the curves intersect at the price \hat{q} , as they do for the lowest supply curve in the figure. In this case, the entrepreneur is indifferent with respect to investment, and in equilibrium, he invests exactly the amount that can be continued at date 1 using the claims issued by the banker at this price. The investment of the entrepreneur is $\omega^e \chi / \left[1 + \hat{q} (\rho - \tilde{R}^e) \right]$, where χ is given by

$$\frac{\omega^b \tilde{R}^b}{1 - \hat{q} \tilde{R}^b} = \frac{\omega^e (\rho - \tilde{R}^e)}{1 + \hat{q} (\rho - \tilde{R}^e)} \chi,$$

the equation of the liquidity supplied by the banker to the demand of the entrepreneur.

Example 1. The parameters used in the plot are $R^b = \frac{5}{4}$, $\gamma^b = 1$, $R^e = 3$, $\gamma^e = 2$, and $\rho = \frac{3}{2}$. One can calculate that $\tilde{R}^b = \frac{1}{4}$, $\tilde{R}^e = 1$, and $\hat{q} = 2$. The endowment of the entrepreneur is $\omega^e = 1$, and the endowments of the banker for the three curves

are $\omega^b = \frac{5}{4}, \frac{3}{4}, \frac{1}{4}$.

In the $\phi = 1$ case, the entrepreneur may be more precisely described as an agent in need of means of preserving the continuity of a project that is valuable only to him privately. In particular, there is no attraction in the project for an outside speculator. A suitable allegory is perhaps that of a buyer of a house at date 0. At date 1, after he has retired, the roof will need to be repaired. At date 2, the house can be sold if it is properly maintained, but the house buyer will not maintain it if a high proportion of sale proceeds must be allocated to repay a loan taken at date 1 to pay for the roof. Therefore the required capital cannot simply be raised ex post by placing a lien on the house.

The problem of this entrepreneur is to buy the biggest house possible while storing enough of his date 0 wealth to meet the date 1 obligation. If there is a sufficient quantity of real assets in the economy at date 0, then the problem is a simple one based on fair accounting at fundamentals. On the other hand, if such assets are scarce, then the degree of this scarcity will affect the entrepreneur's decisions in a non-fundamental way.

The entrepreneur's demand for liquidity is met by the issue by the banker of liquid liabilities. Considering the banker's balance sheet at date 0, one finds assets consisting of his illiquid investment, and liabilities consisting of claims against the

yield of that investment. One may interpret these liabilities as having a two-period maturity, rather than literally being redeemed for an equivalent issue. Under this interpretation, some of these privately issued claims circulate from the entrepreneur to whom they are sold at date 0 to the worker at date 1, by whom they are finally redeemed at date 2. These liabilities behave as money in a well-defined sense. ■

3.3 More General Cases

The generalization of the model introduces important conceptual nuances to be explored in this subsection. Because of the richness of possibilities and the heterogeneity of agents' opportunities, this discussion is potentially very complex. For this reason, I will assume in the remainder of the text that the probability of the liquidity shock is low enough that the entrepreneur's project may be profitably undertaken even when it will be fully liquidated in the bad state. This assumption is not necessary, but it simplifies the exposition without significant conceptual loss.

From the previous subsection, one can get the flavor of the case excluded here. When ϕ is high and as the price of liquidity rises, the entrepreneur will prefer to reduce his a priori investment rather than liquidate his project after it is begun. When this probability is low, on the other hand, he will do the opposite; that is, he will invest as much as possible and liquidate if necessary. In the market for liquidity,

the two modes of behavior are qualitatively identical, since the liquid claims desired by the entrepreneur will be proportional to the product of his investment and the fraction of the project that he will continue, $I^e (1 - \lambda_H^e)$. Because one case holds little interest independent of the alternative, I choose to relegate one to an appendix to simplify matters. Formally, I assume in the remainder of the text that

$$(1 - \phi) R^e - 1 > 0. \tag{22}$$

With respect to investment at date 0, the motivation of the banker is little changed in the general environment from that of the special case considered above. The restrictions derived on equilibrium prices in subsection 3.1 imply that the private rate of return available from investing will always exceed the market rate of interest. Therefore the banker will invest all of his endowment in his project, and he will arrange his portfolio in order to maximize expected consumption at date 2. In particular, he will forego consumption at dates 0 and 1, and leverage date 2 consumption maximally so that incentive compatibility constraints bind. The following lemma contains the formal statement of this fact; the proof is contained in an appendix.

Lemma 5 *At equilibrium prices, the banker will choose to consume nothing at dates*

0 and 1, and he will choose date 2 consumption so that his incentive compatibility constraint is binding in each state; that is, $c_0^b = 0$, and $c_{1s}^b = 0$ and $c_{2s}^b = (1 - \lambda_s^b) I^b \gamma^b$ for each s .

In the case of the entrepreneur, the simplification discussed above is immediately relevant and apparent. Recall from the special case $\phi = 1$ that the entrepreneur may be indifferent about investing when the price of liquidity becomes too high. In sharp contrast to the banker's behavior, it was shown that the entrepreneur may even choose to consume some of his endowment rather than invest it. The next lemma shows that this is not the case when ϕ is low; the proof is contained in an appendix.

Lemma 6 *Suppose that (22) holds. At equilibrium prices, the entrepreneur will choose to consume nothing at dates 0 and 1, and he will choose date 2 consumption so that his incentive compatibility constraint is binding in each state; that is, $c_0^e = 0$, and $c_{1s}^e = 0$ and $c_{2s}^e = (1 - \lambda_s^e) I^e \gamma^e$ for each s .*

At an optimum under equilibrium prices q , Lemma 5 and the binding budget constraints of the banker's problem can be seen to yield

$$I^b = \frac{\omega^b}{1 - \sum_{s \in S} q_{1s} \left[\lambda_s^b L + q_{2s} (1 - \lambda_s^b) \tilde{R}^b \right]}. \quad (23)$$

Generalizing the argument put forth for the simpler case, it is easy to see that the

denominator of this expression must be positive at equilibrium prices for all feasible liquidation rules of the banker. If this were not the case, inspection of the form of the problem shows that a feasible policy exists that gives the banker any arbitrarily large payoff. This is obviously precluded by the definition of equilibrium. As before, the intuition for why this cannot hold in equilibrium is that the liquid claims issued by the banker will be in proportion to his payoff, so that finite demand for these claims can be met by the supply of the banker at a price lower than that affording him infinite payoff.

Now using Lemma 5 and eliminating investment using (23), some algebra shows that the optimal liquidation rule for the banker maximizes

$$\gamma^b \left\{ \frac{\omega^b \sum_s \phi_s (1 - \lambda_s^b)}{1 - \sum_s q_{1s} [\lambda_s^b L + q_{2s} (1 - \lambda_s^b) \tilde{R}^b]} \right\}. \quad (24)$$

This objective function has a simple interpretation. The factor γ^b is the banker's date 2 consumption per unit of residual (i.e., un-liquidated) investment at that date. The expression in braces is the average (i.e., expectation) over states of the residual investment, which can usefully be dissected further as follows. The factor

$$\kappa^b := \left\{ 1 - \sum_s q_{1s} [\lambda_s^b L + q_{2s} (1 - \lambda_s^b) \tilde{R}^b] \right\}^{-1}$$

is the maximal ratio of investment to the banker's initial endowment that is afforded by the incentive compatibility constraints, which is the *leverage ratio* of the banker. Therefore $\omega^b \kappa^b$ is the investment of the entrepreneur. The last factor, $\sum_s \phi_s (1 - \lambda_s^e)$, is clearly the unconditional expectation of the fraction of the banker's project that will not be liquidated before date 2 under the chosen policy. Obviously, the banker gets no utility from liquidating a portion of his project.

Under condition (22), the same logic can be applied to the case of the entrepreneur to show that his investment is⁸

$$I^e = \frac{\omega^e}{1 - \sum_s q_{1s} (1 - \lambda_s^e) (q_{2s} \tilde{R}^e - \rho_s)}, \quad (25)$$

and his liquidation decision boils down to choosing $\lambda^e \in [0, 1]^2$ to maximize

$$\gamma^e \left\{ \frac{\omega^e \sum_s \phi_s (1 - \lambda_s^e)}{1 - \sum_s q_{1s} (1 - \lambda_s^e) (q_{2s} \tilde{R}^e - \rho_s)} \right\}.$$

The latter admits an interpretation similar to that for (24).

Although it is straightforward to characterize the optimal liquidation policies at this point by jumping to the first-order Kuhn-Tucker conditions of these strictly

⁸Analogous to the case of the banker, in order for the entrepreneur's problem to have a solution, the denominator of the expression on the RHS must be positive for all $\lambda^e \in [0, 1]^2$; therefore, equilibrium prices must have this property.

quasiconcave functions, I want first to show how the price space may be simplified. Doing so drastically simplifies the manipulation and intuition value of these characterizations.

Corollary 7 *At equilibrium prices, it must be that $q_{1L} = 1 - \phi$ and $q_{2s} = 1$ for each s .*

The proof, which is contained in an appendix, proceeds by showing that the extra-fundamental prices in any market except that for state $1H$ claims must either violate market clearing or the result of Proposition 2. In particular, in each market except that for claims in state $1H$, both the banker and the entrepreneur will be net issuers (negative claims holders). Therefore, the relevant market clearing conditions imply that the claims holdings of the worker must be positive for these markets, and then Proposition 2 shows that the fundamental prices must prevail.

It is now possible to characterize the equilibrium liquidation policies followed by the agents in the most simple form. Thereafter I can characterize the equilibrium with reference only to the market for liquidity (that is, state $1H$ claims), by pinning down the price of liquidity q_{1H} .

For the banker, one has the following.

Lemma 8 *Suppose that q is an equilibrium price system.*

1. If $\tilde{R}^b \geq L$ then the banker will never liquidate his project in any state in equilibrium.
2. If $\tilde{R}^b < L$, then, in equilibrium, the banker will never liquidate his project in the good state, but liquidation will occur in the bad state if the price of state 1H claims is high enough. More precisely, the optimal liquidation policy for the banker has $\lambda_L^b = 0$ and $\lambda_H^b \in \Lambda^b(q_{1H})$, where

$$\Lambda^b q_{1H} := \begin{cases} 0, & \text{if } q_{1H} < q^b \\ [0, 1], & \text{if } q_{1H} = q^b \\ 1, & \text{if } q_{1H} > q^b, \end{cases}$$

and

$$q^b := \frac{1 - (1 - \phi) \tilde{R}^b}{L - (1 - \phi) \tilde{R}^b} \phi.$$

It may be surprising that the banker would ever liquidate his project in this environment, since he can always choose his investment and structure his claims portfolio so that it is not necessary to do so. The key to the seeming paradox obviously lies in the condition that the marketable share of the banker's project must be less than its liquidation value for liquidation to be optimal. In this case, the banker can raise more outside funds by promising to liquidate. Therefore, though the banker

gets no payoff in the event, liquidation in the bad state allows him to increase his investment. If the price of liquidity is high enough, the increased investment afforded may sufficiently increase his payoff in the good state to compensate (in expectation) for the sacrifice of any payoff in the bad state. This result will be discussed at greater length in the next section.

For the entrepreneur, the optimal liquidation policy abides the following.

Lemma 9 *Assume that condition (22) holds. At equilibrium prices, the entrepreneur will never liquidate his project in the good state, and there is a cutoff level $\bar{q} > \phi$ of the price of liquidity such that liquidation is optimal in the bad state if and only if $q_{1H} \geq \bar{q}$. More precisely, optimality has $\lambda_L^e = 0$ and $\lambda_H^e \in \Lambda^e(q_{1H})$, where*

$$\Lambda^e(q_{1H}) := \begin{cases} 0, & \text{if } q_{1H} < \bar{q} \\ [0, 1], & \text{if } q_{1H} = \bar{q} \\ 1, & \text{if } q_{1H} > \bar{q}, \end{cases}$$

and

$$\bar{q} := \frac{1 - (1 - \phi) \tilde{R}^e}{(1 - \phi) (\rho - \tilde{R}^e)} \phi.$$

Lemmas 8 and 9 are proved in the appendix.

Writing the investment of the banker under equilibrium prices q as

$$I^b = \mathcal{I}^b(q_{1H}, \lambda^b) := \frac{\omega^b}{1 - (1 - \phi) \tilde{R}^b - q_{1H} [\lambda_H^b L + (1 - \lambda_H^b) \tilde{R}^b]},$$

the banker's claims holding can be summarized as follows.

Proposition 10 *Suppose that q is an equilibrium price system.*

1. *The banker will be a net issuer of claims in each state; that is, his holdings of claims will be non-positive.*
2. *If the liquidation value of the banker's project is no greater than its marketable share, then his holdings of claims will be strictly negative and equal in each state; precisely, if $L \leq \tilde{R}^b$, then*

$$B_{ts}^b = -\mathcal{I}^b(q_{1H}, 0) \tilde{R}^b = -\frac{\omega^b \tilde{R}^b}{1 - (1 - \phi + q_{1H}) \tilde{R}^b}$$

for $t \in \{0, 1\}$ and $s \in \{H, L\}$.

3. *If the liquidation value of the banker's project is greater than its marketable share, then (i) his holdings of claims will be negative in state 1s for each s , and in state 2L; but (ii) his state 2H holdings may reflect liquidation of a portion of his project and retirement of liabilities. More precisely, there is an optimal*

policy for the banker with claims holdings B^b if and only if

$$\begin{aligned} B_{1L}^b &= B_{2L}^b = -\mathcal{I}^b(q_{1H}, \lambda_H^b) \tilde{R}^b \\ B_{1H}^b &= -\mathcal{I}^b(q_{1H}, \lambda_H^b) \left[\lambda_H^b L + (1 - \lambda_H^b) \tilde{R}^b \right] \\ B_{2H}^b &= -\mathcal{I}^b(q_{1H}, \lambda_H^b) (1 - \lambda_H^b) \tilde{R}^b, \end{aligned}$$

for some $\lambda_H^b \in \Lambda^b(q_{1H})$.

Proof. The result follows easily from the banker's budget constraints in light of previous results. ■

The following proposition characterizes the set of state $1H$ claims holdings that can be optimal for the entrepreneur at equilibrium prices. This knowledge is all one needs to know about the behavior of the entrepreneur in order to characterize the equilibrium prices. The optimal state $1H$ claims holdings of the entrepreneur define his "demand for liquidity".

Proposition 11 *Suppose that (22) holds. At equilibrium prices, there is an optimal policy for the entrepreneur with claims holdings B^e only if*

$$B_{1H}^e = \beta^e(q_{1H}, \lambda_H^e) := \frac{\omega^e (1 - \lambda_H^e) (\rho - \tilde{R}^e)}{1 - (1 - \phi) \tilde{R}^e + q_{1H} (1 - \lambda_H^e) (\rho - \tilde{R}^e)}$$

for some $\lambda_H^e \in \Lambda^e(q_{1H})$.

Proof. The result follows easily from the entrepreneur's budget constraints, equation (25), and Corollary 7. ■

Now the price of liquidity, the only element of the equilibrium price system that has not been pinned down, may be characterized according to the algorithm described in the following proposition. In stating the result, I write $\beta^b(q_{1H}, \lambda_H^b)$ for the banker's issue of claims when the equilibrium price is q_{1H} and his optimal liquidation decision specifies that he liquidate the share λ_H^b of his project in the bad state. Then by Proposition 10,

$$\beta^b(q_{1H}, \lambda_H^e) := \frac{\omega^b [\lambda_H^b L + (1 - \lambda_H^b) \tilde{R}^b]}{1 - (1 - \phi) \tilde{R}^b + q_{1H} [\lambda_H^b L + (1 - \lambda_H^b) \tilde{R}^b]}.$$

Proposition 12 *Suppose that (22) holds.*

1. *If the banker's issue of liquid claims is at least as great as the entrepreneur's demand for them at the fundamental price, then this price supports an equilibrium without liquidation by the entrepreneur. More precisely, if $\beta^b(\phi, 0) + \beta^e(\phi, 0) \leq 0$, then in equilibrium $q_{1H} = \phi$; there is no liquidation; and agents' state 1H claims are given by $B_{1H}^b = \beta^b(\phi, 0)$, $B_{1H}^e = \beta^e(\phi, 0)$, and $B_{1H}^w = -B_{1H}^b - B_{1H}^e$.*
2. *If the banker's issue of liquid claims is less than the entrepreneur's demand*

for them at the fundamental price, and the marketable share of the banker's project is at least as great as its liquidation value, then the equilibrium price of liquidity will be greater than the fundamental price, and the entrepreneur may be required to liquidate a portion of his project in the bad state. Precisely, if $\beta^b(\phi, 0) + \beta^e(\phi, 0) > 0$ and $\tilde{R}^b \geq L$, then q_{1H} uniquely satisfies $\beta^b(q_{1H}, 0) + \beta^e(q_{1H}, \lambda_H^e) = 0$ with $\lambda_H^e \in \Lambda^e(q_{1H})$.

3. If the banker's issue of liquid claims is less than the entrepreneur's demand for them at the fundamental price, and the marketable share of the bankers project is less than its liquidation value, then the equilibrium price of liquidity will be greater than the fundamental price, and one or both of the producers may choose to liquidate a portion of his project in the bad state. Precisely, if $\beta^b(\phi, 0) + \beta^e(\phi, 0) > 0$ and $\tilde{R}^b < L$, then q_{1H} uniquely satisfies $\beta^b(q_{1H}, \lambda_H^b) + \beta^e(q_{1H}, \lambda_H^e) = 0$ with $\lambda_H^b \in \Lambda^b(q_{1H})$ and $\lambda_H^e \in \Lambda^e(q_{1H})$.

Proof. The result follows from the previous ones and the definition of equilibrium.

■

Two additional examples are presented in the next section.

4 Discussion

4.1 Liquidity Provision, Inside Money, and Output

In the model presented here, a liquidity need is generated by the confluence of two factors. First, moral hazard induces a wedge between market and private valuations of projects. Second, inalienable labor implies that unsecured promises may not be credible, so that futures (e.g., insurance) contracts that could otherwise be useful in mitigating the first problem may be impossible to construct. In such an environment, marketable securities may acquire an extra-fundamental value commensurate with their name: they act to secure access to capital in the future by preserving wealth over time.

The liquidity value of collateral securities has been investigated in varied environments by Holmström and Tirole (1998) and Kiyotaki and Moore (1998, 2000, 2001). Holmström and Tirole, whose modeling devices I have essentially incorporated here, investigate (inter alia) the utility of a government bond to ameliorate the liquidity problem. In their model, the supply of the liquid security is exogenous and perfectly elastic. In Kiyotaki and Moore (1998, 2000, 2001), collateral is in fixed (perfectly inelastic) supply. Thus, each of these papers abstracts from the topic of primary interest here.

While each of these papers examines how liquidity problems affect firms that experience them, the important innovation of the present model is a detailed theoretical analysis of how liquidity may be provided by the issue of liquid securities by firms that do not. One implication is that, as long as some firms are unaffected, and as long as those firms are capable of issuing fungible securities, liquidity problems can have qualitatively different effects in different sectors. The following example illuminates the possibility that some sector may be benefited by such episodes.

Example 2. Let $R^b = \frac{12}{7}$, $R^e = 2$, $\gamma^b = \gamma^e = \frac{8}{7}$, $\rho = \frac{8}{7}$, $\phi = \frac{1}{4}$, $\omega^b = \frac{2}{7}$, $\omega^e = \frac{5}{7}$, and suppose that $L \leq \frac{4}{7}$. Thus, $\tilde{R}^b = \frac{4}{7}$, $\tilde{R}^e = \frac{6}{7}$, and $\rho - \tilde{R}^e = \frac{2}{7}$. Note that (22) holds (the left-hand side equals $\frac{1}{2}$), and one can calculate that $\bar{q} = \frac{5}{12}$.

At the fundamental prices, (23) and Lemma 8 (part 1) show that the banker would invest

$$\frac{2/7}{1 - 4/7} = \frac{2}{3}$$

in his project. From Proposition 10 (part 2), it can be seen that his investment would be financed by his issue of $\frac{2}{3} \cdot \frac{4}{7} = 0.381$ claims for each state. At these prices, (25) and Lemma 9 show that the entrepreneur would choose to invest

$$\frac{5/7}{1 - \frac{3}{4} \left(\frac{6}{7} \right) + \frac{1}{4} \left(\frac{2}{7} \right)} = \frac{5}{3}$$

in his project. To finance his investment, Proposition 11 shows that he would like to buy the net amount $\frac{5}{3} \cdot \frac{2}{7} = 0.476$ claims for the bad state.

In the present example, the amount of claims desired by the entrepreneur exceeds the amount that would be issued by the banker at the fundamental prices. Since the worker is precluded from issuing claims by his inability to keep the promise, the market equilibrium will therefore reflect a premium price on liquid claims. Precisely, Proposition 12 (part 2) shows that the market will clear at the price q_{1H} satisfying

$$\begin{aligned} -\beta^b(q_{1H}, 0) &= \frac{\frac{2}{7} \cdot \frac{4}{7}}{1 - \frac{3}{4} \left(\frac{4}{7}\right) - q_{1H} \left(\frac{4}{7}\right)} = \frac{8}{28(1 - q_{1H})} \\ = \beta^e(q_{1H}, \lambda_H^e) &= \frac{\frac{5}{7}(1 - \lambda_H^e) \left(\frac{2}{7}\right)}{1 - \frac{3}{4} \left(\frac{6}{7}\right) + q_{1H}(1 - \lambda_H^e) \left(\frac{2}{7}\right)} = \frac{20(1 - \lambda_H^e)}{35 + 28q_{1H}(1 - \lambda_H^e)} \end{aligned}$$

for some λ_H^e such that

$$\lambda_H^e \in \begin{cases} 0, & \text{if } q_{1H} \in \left[\frac{1}{4}, \frac{5}{12}\right] \\ [0, 1], & \text{if } q_{1H} = \frac{5}{12} \\ 1, & \text{if } q_{1H} > \frac{5}{12}. \end{cases}$$

The solution has $\lambda_H^e = 0$ and $q_{1H} = \frac{5}{14}$. The supply and demand curves for this market are depicted in Figure 2, where the broken curve is $\beta^b(\cdot, 0)$, and the solid curve is $\beta^e(\cdot, \lambda_H^e)$ for $\lambda_H^e \in \Lambda^e(q_{1H})$.

In the equilibrium, the entrepreneur's investment can be seen to be $\frac{14}{9}$, less than

he would choose in an environment with surplus liquidity. On the other hand, the liquidity price-premium represents an implicit subsidy for the investment of the banker. The latter invests more $\left(\frac{7}{9}\right)$ and issues more claims $\left(\frac{4}{9}\right)$ to goods in each state. Thus, the example obviates an interesting feature of the model: that the liquidity problem may distort the banker's behavior in the direction of *increased* investment. ■

Although the investment that backs the securities in the model is illiquid, the securities themselves are fully liquid in the sense that they are marketable. Indeed, the timing of project maturities necessitates the circulation of liquid assets among agents to effect the desired transactions. This emphasizes that the securities need not have short maturities cycles to be liquid. With respect to the real world, of course, there are other properties that affect the "marketability" of securities, and the degree to which the analogous securities are liquid is an empirical question.

It is worth noting that the introduction of fiat money in some extension would not solve the liquidity problem if it earned a rate of return below that earned by real investment, as it would, for example, in a simple model in which agents that discount the future are endowed with a fixed stock of currency. This is because the assets that provide liquidity are just as liquid as fiat money, and earn a better return. But if the frequency of liquidity shocks is similar to the maturity cycle of investment, it seems likely that financially sophisticated firms would not find access to securities markets

inconvenient (since they may access such markets for investment finance, anyway), so that the monetary instrument of choice for meeting these liquidity needs is more likely to be savings deposits, CDs, T-bills, or even corporate bonds. That is, the assumption that such instruments are sufficiently liquid to serve this function is plausible.

This argument suggests that the phenomenon described here relates specifically to the effect of interest-bearing inside money, rather than fiat money, on economic performance. This observation may, in turn, motivate an explanation of an effect of interest-bearing inside money on output. Indeed, evidence suggests that broad measures of money are more highly correlated with output than M1 or the monetary base. Moreover, the finding by Friedman and Kuttner (1993) that the six-month commercial paper rate "is superior [to the three-month T-bill rate] in capturing the information in financial prices that matters for the determination of real income" may be taken as supportive of the idea that it is the whole stock of liquid liabilities, more than just those of the government, that provide important liquidity services.

4.2 Equilibrium Liquidation of the Banker's Assets

In the previous section, I showed that the banker may choose to liquidate his project under certain circumstances, but the intuition for this result may not be

immediately obvious. Nevertheless, I will argue that the rationale is intuitive and economically interesting.

In the model, liquidation of a project circumvents the need to provide incentives. Therefore, even though the overall return from the project is lower when it is liquidated, it is still possible that liquidation may put more value in the hands of outside claims holders than could be achieved by carrying the project through. Indeed, Proposition 10 shows that this is precisely the necessary and sufficient condition for the banker to be willing to liquidate when the price of liquidity becomes extreme. In such an environment, his private objective of expected payoff maximization is served by selling state $1H$ claims based on the liquidation value of his investment, rather than their value at maturity.

My last example illustrates this phenomenon numerically.

Example 3. Let us reconsider the example of the previous subsection with the simple change that now $L = \frac{6}{7} > \tilde{R}^b$. In this case, Lemma 8 shows that liquidation will be optimal for the banker whenever q_{1H} is at least $q^b = \frac{1}{3}$.

Figure 3 depicts the demand correspondence of the entrepreneur, analyzed in Example 2, together with the implied supply correspondence of the banker with the higher liquidation value. As is apparent from the figure, the new equilibrium price is $\frac{1}{3}$, which is lower than it was in the previous example, and the banker now chooses to

liquidate a small fraction of his project in the bad state. More precisely, the fraction of the banker's project to be liquidated in equilibrium satisfies

$$\begin{aligned} -\beta^b \left(\frac{1}{3}, \lambda_H^b \right) &= \frac{\frac{2}{7} [\lambda_H^b (\frac{6}{7}) + (1 - \lambda_H^b) (\frac{4}{7})]}{1 - \frac{3}{4} (\frac{4}{7}) - \frac{1}{3} [\lambda_H^b (\frac{6}{7}) + (1 - \lambda_H^b) (\frac{4}{7})]} \\ &= \beta^e \left(\frac{1}{3}, 0 \right) = \frac{20}{35 + 28 (\frac{1}{3})}, \end{aligned}$$

and it may be calculated that $\lambda_H^b = \frac{2}{29}$. ■

Documenting the S&L crisis in the U.S., White (1989) and others have observed banks in financial distress "gambling for resurrection" by choosing to undertake risky projects when their prospects are already low. And Dewatripont and Tirole (1994) and other authors have adequately explained the phenomenon theoretically. While such behavior may appear to be related to that described here, closer observation reveals that is clearly quite different. In particular, the environment described by these authors pre-supposes private information possessed by the bank about the quality of its portfolio prior to the investment decision. Gambling by the bank is then adverse to the interest of the holders of the bank's liabilities. In the world described here, there is perfect information, and liquidation, when it occurs, represents a constrained optimum.

A more plausible historical analogy is that to episodes of liquidation by institu-

tions under the National Banking System of the nineteenth century. In that era, liquidity crises were relatively frequent occurrences, and liquidations by banks often coincided with them. This was true even while the crises usually did not directly affect the banks' assets. The closure of the analogy implies that, in times of the most severe crises, banks' liabilities did not garner sufficient faith that they could be circulated, and redemptions necessitated asset liquidations. But given the frequency and systematic nature of such occurrences, it defies belief that the gamble by banks was not undertaken consciously and widely understood. That is, this failure of faith must have been expected *ex ante*.

4.3 Bank Capital and Liquidity

To the extent that the endowment of the banker is closely linked to the concept of bank capital, my model alludes to a micro-foundation for an effect of bank capital for the effectiveness of the financial system. That is, the productive capacity if the entrepreneur is affected in a coherent way by the availability of bank liabilities; and, for a fixed equilibrium interest rate, the issue of monetary liabilities by the banker is greater in proportion to this "bank capital".

This may be seen quite simply with reference to the first example in subsection 3.2. There the effect of increasing the endowment "capital" of the bank is clearly

exemplified by the increase in the equilibrium interest rate (decrease in the price of liquidity) as the supply of liquidity by the banker is shifted up.

Unfortunately, the present model can do no more than hint at an interesting connection, and no extension suitable for investigating it will be undertaken here.

4.4 Implications for Central Bank Policy

Although I have argued above that the introduction of fiat money would likely be of little value in mitigating the type of problem I have discussed, it does not automatically follow that there can be no scope for intervention by a government authority here. As usual, this issue is sensitive to the attributes ascribed to this entity by the theory.

It will be easily recognized that liquidity problems may be eased to the extent that the government can issue liabilities at date 0 that will be valuable to the worker at date 1; in effect, the ability of the government to commit to such a plan coupled with the capacity to act on it will be valuable. This discussion naturally leads one to ponder the importance of the issue and redemption of bonds by the government. Aspects of this issue are addressed by Holmström and Tirole (1998). But the stock of bonds issued by the government represents a fraction of the entire stock of liquid assets in the economy, and it is likely that some of these are suitable substitutes

for government bonds in many circumstances. As such, it is unsatisfying for some purposes to imagine simply that liquid assets are available at a price that is fixed exogenously.

Alternatively, the government could affect aggregate liquidity merely by its commitment to affect the total state $1H$ value of the stock of liquid assets that are available at date 0. This idea suggests some value for open market operations in this environment; that is, the government might be imagined to increase the value of this stock by purchasing assets in this state. By analogy, the theory described here suggests that open market purchases will increase liquidity by enhancing the quality of the balance sheets of agents who hold these assets. Note that this story is fundamentally different from the money- or credit-channel models of the effectiveness of monetary policy, and it does not involve fiat money or any "special" status of banks as such. In this way, it is more in line with the "bank capital channel" story suggested by Van den Heuvel (2002).

In any case, it is clear that study of these issues requires one to take a stand on exactly what powers the central authority has that agents do not. I will conclude this paper without having done so.

5 Conclusions

In this paper, I have presented a model of entrepreneurial finance by heterogeneous producers in general equilibrium. Entrepreneurs issue securities in order to finance an initial investment, but they must also make arrangements to insure against a possible liquidity shock. Moral hazard on the part of the entrepreneur necessitates that he retain a share of the project so that he has an incentive not to abuse his position. But ensuring access to capital at the necessary time is hindered, in addition, by the lack of commitment on the part of other agents. Since the moral hazard problem remains even after the liquidity shock, an uninsured entrepreneur may be forced to abandon a project that has positive economic value. The lack of commitment by outsiders implies that they may be unwilling ex post to make good on promises exchanged for securities that turn out to have low value.

To mitigate the problem, the entrepreneur will desire to hold securities that have high value ex post. Thus, securities that have value in the bad state provide liquidity services owing to market frictions, and they may merit a price premium ex ante.

It seems natural to try to interpret the issuers of securities that provide liquidity services as "banks". This interpretation leads to the conclusion that broad definitions of "money" – in particular, interest-bearing liabilities – may be a powerful determinant of the liquidity of an economy. This interpretation seems to be consistent with

the stylized fact that broader definitions of money are more highly correlated with output than the monetary base.

But it is important to recognize a complementary interpretation that suggests that the stock of assets that provide liquidity services may be broader still. That is, there is nothing in the model that should lead one to associate the liquid liabilities of the banker more with bank deposits than with CDs or even the bonds of non-financial corporations. An investigation contrasting the role of these instruments for the provision of liquidity seems an interesting avenue for future research.

Though the mechanism by which liquidity becomes scarce has been adopted in its essence from Holmström and Tirole (1998), this paper contributes to the extant theory by recognizing how the introduction of heterogeneity changes the environment fundamentally. In particular, the innovation allows the use by agents of private liabilities backed by investment for their liquidity needs. In turn, this implies that the stock of liquid assets inherits the elasticity properties of the investment itself. In this environment, one may interpret the endogenous emergence of "banks" in a simple and fully-specified perfect-information economy. I believe that the important properties of the model can be extended in a straightforward way to a dynamic economy, facilitating the study of the interaction between financial and real business cycles.

6 Proofs of the Results

Proof of Lemma 5. First suppose that $(c^b, I^b, \lambda^b, B^b)$ is a feasible policy, and suppose that $c_0^b = \Delta > 0$. Now construct the alternative policy $(\tilde{c}^b, \tilde{I}^b, \tilde{\lambda}^b, B^b)$ as follows. Let $\tilde{c}_0^b = 0$, $\tilde{I}^b = I^b + \Delta$, and $\tilde{\lambda}_s^b = \lambda_s^b I^b / \tilde{I}^b$ and $\tilde{c}_{2s}^b = c_{2s}^b + \Delta R^b$ for each s . Let the remaining elements of the new policy be identical to the old. Now it is easy to check that the new policy is feasible if the old one is. Moreover, subtracting the payoff under the old policy from that generated by the new gives $\Delta (R^b - 1) > 0$, so that the new one is an improvement, a contradiction.

To see that the banker's incentive compatibility constraints must bind, let $(c^b, I^b, \lambda^b, B^b)$ be a feasible policy for the banker, and suppose that

$$c_{2\sigma}^b = I^b (1 - \lambda_\sigma^b) \gamma^b + \Delta$$

for some $\Delta > 0$. Now construct the alternative policy $(\tilde{c}^b, \tilde{I}^b, \tilde{\lambda}^b, B^b)$ as follows. Let $\tilde{I}^b = I^b + q_{1\sigma} q_{2\sigma} \Delta$, and $\tilde{\lambda}_s^b = \lambda_s^b I^b / \tilde{I}^b$ for each s ; and let $\tilde{c}_{2\sigma}^b = c_{2\sigma}^b - \Delta + q_{1\sigma} q_{2\sigma} \Delta R^b$ and $\tilde{c}_{2s}^b = c_{2s}^b + q_{1\sigma} q_{2\sigma} \Delta R^b$ for $s \neq \sigma$. Let the remaining elements of the new policy be identical to the old. Now it is easy to check (using $q_{1\sigma} q_{2\sigma} \geq \phi_\sigma$) that the new policy is feasible if the old one is. Moreover, subtracting the payoff under the old

policy from that generated by the new gives

$$\Delta (q_{1\sigma} q_{2\sigma} R^b - \phi_\sigma) \geq \Delta \phi_\sigma (R^b - 1) > 0.$$

Finally, suppose that the feasible policy has $c_{1\sigma}^b = \Delta > 0$. Then construct the tilde policy with $\tilde{c}_{1\sigma}^b = 0$, $\tilde{I}^b = I^b + q_{1\sigma} \Delta$, $\tilde{\lambda}_s^b = \lambda_s^b I^b / \tilde{I}^b$, and $\tilde{c}_{2s}^b = c_{2s}^b + q_{1\sigma} R^b$; and let the other elements be as in the original policy. Again the feasibility and superiority of the new policy can be verified. ■

Proof of Lemma 6. Suppose that $(c^e, I^e, \lambda^e, B^e)$ is a feasible policy, and that $c_0^e = \Delta > 0$. Construct an alternative policy $(\tilde{c}^e, \tilde{I}^e, \tilde{\lambda}^e, B^e)$ as follows. Let $\tilde{c}_0^e = 0$, $\tilde{c}_{2L}^e = c_{2L}^e + \Delta R^e$, $\tilde{I}^e = I^e + \Delta$, $\tilde{\lambda}_L^e = I^e \lambda_L^e / \tilde{I}^e$, and $\tilde{\lambda}_H^e = 1 - (1 - \lambda_H^e) I^e / \tilde{I}^e$. Define the remaining elements of the new policy as in the old. Now it is easy to check that the new policy is feasible if the old one is. Moreover, subtracting the payoff to the entrepreneur of the old policy from that generated by the new one gives

$$\Delta [(1 - \phi) R^e - 1] > 0,$$

where the inequality follows from (22).

(Surpluses for the other states can be excluded in the manner of the proof of Lemma 5.) ■

Proof of Corollary 7. From the budget constraints of the banker, one has the following expressions for optimal claims holding:

$$\begin{aligned} B_{1s}^b &= -I^b \left[\lambda_s^b L + q_{2s} (1 - \lambda_s^b) \tilde{R}^b \right] < 0 \\ B_{2s}^b &= -I^b (1 - \lambda_s^b) \tilde{R}^b \leq 0. \end{aligned}$$

(The strict inequality follows from the fact that $I^b > 0$.) Suppose by way of contradiction that $q_{1L} > 1 - \phi$. It is immediate from Proposition 2 that $B_{1L}^w = 0$, and from the budget constraints of the entrepreneur and Lemma 6, it can be seen that

$$B_{1L}^e = -q_{2L} I^e (1 - \lambda_L^e) \tilde{R}^e \leq 0.$$

But now $\sum_s B_{1L}^i < 0$, contradicting the market clearing conditions. Thus, it must be that $q_{1L} = 1 - \phi$.

Next suppose that $q_{2s} > 1$; then Proposition 2 gives $B_{2s}^w = 0$, and from the budget constraints of the entrepreneur and Lemma 6, it can be seen that

$$B_{2s}^e = -I^e (1 - \lambda_s^e) \tilde{R}^e \leq 0. \tag{26}$$

Since $B_{2s}^i \leq 0$, $i \in \{b, e\}$, market clearing implies that $B_{2s}^i = 0$ for $i \in \{b, e\}$. Since

$I^b > 0$, this implies that $\lambda_s^b = 1$. From the first-order (n.s.) conditions for (24), it is straightforward to show that $\lambda_s^b > 0$ only if $q_{1\sigma} > \phi_\sigma$ for some $\sigma \in \{H, L\}$.⁹ Suppose first that $\sigma = s$; that is, $q_{1s} > \phi_s$. Then Proposition 2 gives $B_{1s}^w = 0$, so that $B_{1s}^e = -B_{1s}^b > 0$. Now from the budget constraints of the entrepreneur and Lemma 6, one has

$$B_{1s}^e = -q_{2s}I^e(1 - \lambda_s^e)(\tilde{R}^e - \rho_s),$$

and it must be that $I^e(1 - \lambda_s^e) > 0$. But now (26) contradicts the implication shown above that $B_{2s}^e = 0$.

Next suppose that σ is not the same as s . Again Proposition 2 gives that $B_{1\sigma}^w = 0$, and $I^e(1 - \lambda_\sigma^e) > 0$. This implies that the entrepreneur liquidates his project in the good state, which can easily be seen (e.g., from the first-order conditions for the problem) to contradict optimality. ■

Proof of Lemma 8. The necessary and sufficient first-order Kuhn-Tucker conditions for a maximum of the strictly quasiconcave objective function (24) are

$$\begin{aligned} \frac{q_{1\sigma}}{\phi_\sigma} (L - \tilde{R}^b) \sum_s \phi_s (1 - \lambda_s^b) - \left\{ 1 - \sum_s q_{1s} [\lambda_s^b L + (1 - \lambda_s^b) \tilde{R}^b] \right\} &\geq 0 \text{ if } \lambda_\sigma^b > 0 \\ &\leq 0 \text{ if } \lambda_\sigma^b < 1, \end{aligned}$$

⁹Writing the Kuhn-Tucker condition for λ_s^b and plugging in $q_{1\sigma} = \phi_\sigma$ for each σ , the criterion reduces to $L - 1$; this quantity is always negative, implying that $\lambda_s^b = 0$ by the Kuhn-Tucker theorem.

where I have imposed the result of Corollary 7 that $q_{2s} = 1$ in equilibrium. I have already argued that the term in braces must be positive for all $\lambda^b \in [0, 1]^2$ at equilibrium prices, so result 1 is obvious by inspection.

Now consider the case that $\tilde{R}^b < L$. using the result of Corollary 7 that $q_{1L} = 1 - \phi$, the critical condition for $\lambda_L^b > 0$ to be optimal can be written as $q_{1H} \geq Q_L(\lambda_H^b)$ where

$$Q_L(\lambda_H^b) := \frac{1 - (1 - \phi)L - \phi(L - \tilde{R}^b)(1 - \lambda_H^b)}{\lambda_H^b L + (1 - \lambda_H^b)\tilde{R}^b};$$

and that for $\lambda_H^b > 0$ can be written as $q_{1H} \geq Q_H(\lambda_L^b)$ where

$$Q_H(\lambda_L^b) := \frac{\phi \left\{ 1 - (1 - \phi) \left[\lambda_L^b L + (1 - \lambda_L^b) \tilde{R}^b \right] \right\}}{\phi L + (L - \tilde{R}^b)(1 - \phi)(1 - \lambda_L^b)}.$$

It is straightforward to show that $Q_H - Q_L < 0$,¹⁰ so that $q_{1H} \geq Q_L(\lambda_H^b)$ implies that $q_{1H} > Q_H(\lambda_L^b)$. Therefore $q_{1H} \geq Q_L(\lambda_H^b)$ implies that $\lambda_H^b = 1$. But inspection of the relation $q_{1H} \geq Q_L(1)$ reveals a contradiction to the equilibrium condition

$$1 - \sum_s q_{1s} \left[\lambda_s^b L + (1 - \lambda_s^b) \tilde{R}^b \right] > 0.$$

¹⁰To show this, first show that Q_L is decreasing in λ_H^b and Q_H is increasing in λ_L^b . Thus $Q_H(\lambda_L^b) - Q_L(\lambda_H^b) \leq Q_H(1) - Q_L(0)$. Then evaluating the RHS, it is easy to see that it must be negative.

Thus, it cannot be that $\lambda_L^b > 0$ in equilibrium, proving the first part of result 2. Now evaluating $Q_H(0)$, the critical condition for $\lambda_H^b > 0$ can be restated as $q_{1H} \geq q^b$, and the optimality of the rule $\lambda_H^b \in \Lambda^b(q_{1H})$ follows directly. ■

Proof of Lemma 9. The function to be maximized by λ^e is strictly quasiconcave, so that the first-order Kuhn-Tucker conditions are necessary and sufficient for optimality. Using the result that $q_{2s} = 1$ (Corollary 7), the first-order condition for λ_σ^e is

$$\begin{aligned} -\frac{q_{1\sigma}}{\phi_\sigma} \left(\tilde{R}^e - \rho_\sigma \right) \sum_s \phi_s (1 - \lambda_s^e) - \left[1 - \sum_s q_{1s} (1 - \lambda_s^e) \left(\tilde{R}^e - \rho_s \right) \right] &\geq 0 \text{ if } \lambda_\sigma^e > 0 \\ &\leq 0 \text{ if } \lambda_\sigma^e < 1. \end{aligned}$$

I have already argued that the expression in square brackets is positive, so that $\rho_L = 0$ implies immediately that $\lambda_L^e = 0$. Imposing this result in the condition for λ_H^b and using the fact that $q_{1L} = 1 - \phi$ (Corollary 7), the cutoff price \bar{q} may be derived by simplifying and solving for the value of q_{1H} that makes the left-hand side criterion exactly equal to zero. ■

7 Figures for Chapter I

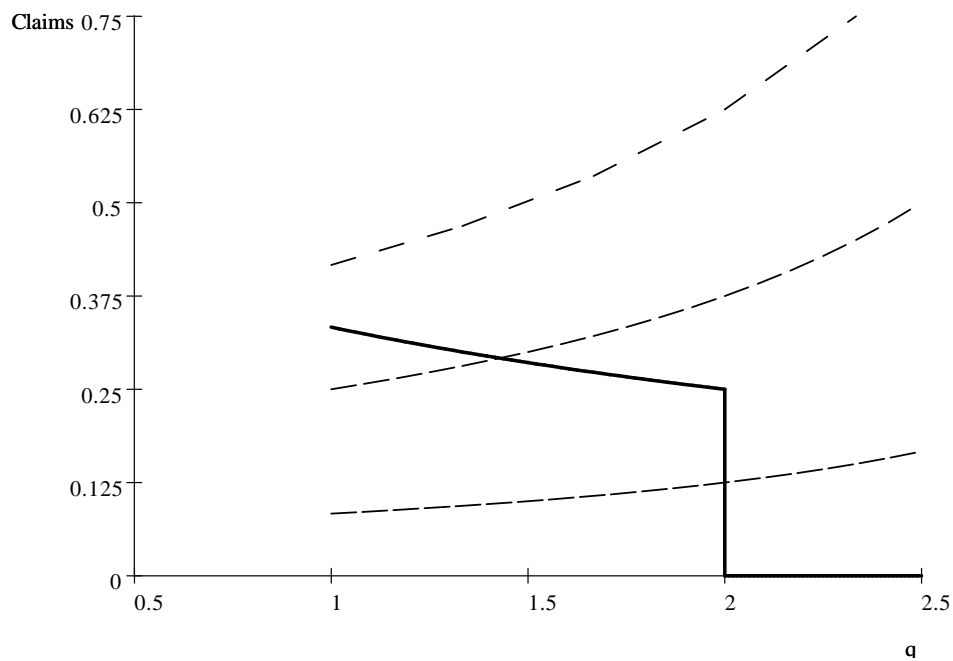


Figure 1. “Supply” and “Demand” of Liquidity for the Parameterization of

Example 1.

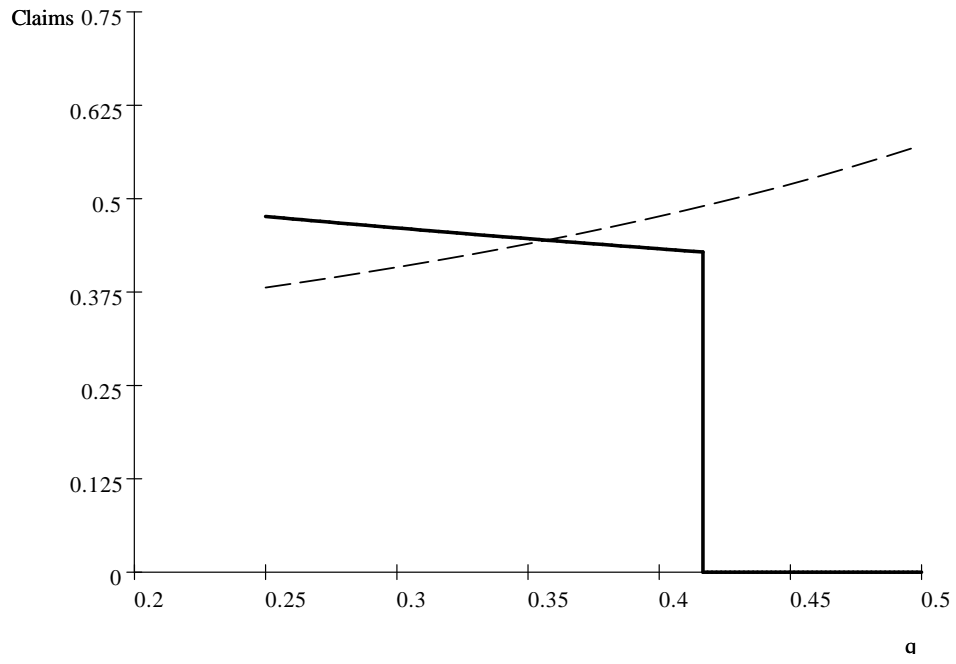


Figure 2. *Supply and Demand of Liquidity for the Parameterization in Example 2.*

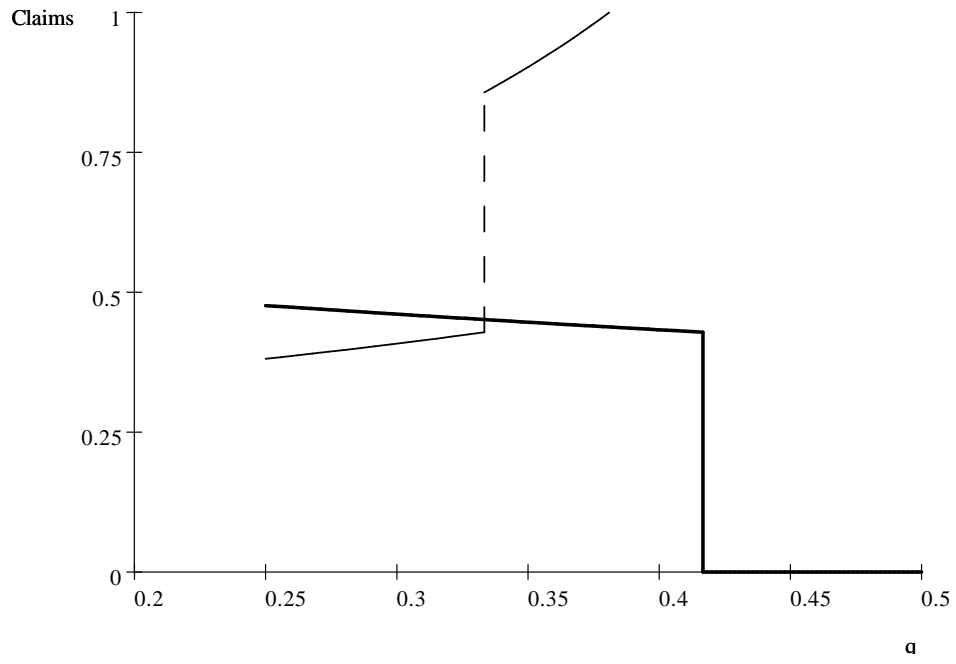


Figure 3. *Supply and Demand of Liquidity in the Parameterization of Example 3.*

Chapter II

Enforcing Contracts by Imperfect Permanent Exclusion from Assets Markets

8 Introduction

The defining characteristic of a collateral asset is that its ownership is contractible, and that the asset is susceptible to confiscation in the event of a breach of contract. In many models of economic equilibrium, the introduction of a collateral asset can be analyzed simply by redefining one or more market clearing conditions to reflect additional resources from "outside".¹¹ This is a simple and intuitively appealing notion, and the effects on the economy are likely to be equally clear and intuitive. But it is not clear from this definition what it would mean for an asset not to be "collateralizable".

The innovation of this paper is a rigorous micro-foundation for the nature of economies with assets that are not collateralizable. I study a one-good, pure-exchange economy with limited enforcement of intertemporal contracts in which defaulting

¹¹In stochastic economies with a complete set of contingent claims markets, for example, collateral can be thought of as augmenting the aggregate stock of claims in each market in a manner conformable state-by-state with the dividend of the asset in that state; that is, the asset is "outside wealth" that affords negative net issue of claims by "insiders".

agents cannot be fully excluded from participation in asset markets. Rather, a tradable non-collateral asset exists that cannot be confiscated, and agents cannot be restricted from trading this asset in spot markets. Thus, while banished from other markets, agents can still self-insure by accumulating and decumulating the non-collateral asset through trade after having reneged on a financial contract.

At the level of quantitative analysis, the assumption, introduced by Kehoe and Levine (1993), that defaulting agents can be excluded entirely from intertemporal trade has proved to be quite restrictive, and a growing branch of the literature has sought to relax it in satisfying ways. Kehoe and Perri (2002) and Seppälä (1999) study economies in which agents can continue to produce and consume capital in "autarky", but they may not buy or sell capital or financial assets. These models capture the fact that agents have alternative ways to smooth their consumption, making life after default less painful than it would be otherwise. Lustig (2001, 2003) assumes that default involves surrender of an agent's holdings of a collateral asset, but that agents may rejoin the assets markets immediately after a "bankruptcy". Thus, Lustig concedes the reality that the worst punishment that can be invented by society is not applied in fact. Like those of Kehoe and Perri and Seppälä, agents in my economy retain a special part of their accumulated portfolio through default; and, like those of Lustig, agents may trade after having defaulted.

I view this paper as drawing also on the literature on consumption smoothing through precautionary saving in economies with incomplete markets. In such settings, Telmer (1993) and Lucas (1994) (and others) have shown that accumulation of a single non-contingent asset in sufficient supply may afford a great deal of consumption smoothing in general equilibrium. This fact suggests that default in the economy I have described may be very attractive. Of course, prices faced by defaulting agents may be different from either those implied by first-best risk-sharing, or general equilibrium with incomplete markets. Rather, prices are determined by institutions that require that default must be (weakly) the less attractive option; if not, then the equilibrium is broken by agents' choice to default.

The equilibration mechanism in the model is capable of inducing a great deal of volatility to the price of the non-collateral asset, and to other endogenous prices and quantities in the model. Intuitively, greater ownership of the non-collateral asset expands an agent's payoff should he choose to default. But this default is, by definition, an out-of-equilibrium occurrence. In the language of the literature on repeated games, the "punishment" for default becomes less harsh with expanded ownership of the non-collateral asset. As in that literature, the net effect can be that the set of actions that can be sustained in an equilibrium is reduced. In the present setting, this means that some contractual promises lose their credibility, enforcement

constraints become tighter, and the scope for unsecured finance is reduced.

To assess the quantitative properties of the model, I follow much of the macro-economic literature in documenting asset pricing implications of a calibrated version of economy. To facilitate comparison with previous work, I adopt the calibration framework of Alvarez and Jermann (2001) and followed also by Lustig (2003). I interpret the non-collateral asset as "equity" in these experiments.¹²

In the next section, I describe the model. The Analysis section described the relevant features of equilibrium and the asset pricing implications. The fourth section described the computational algorithm, the calibration, and the results of my numerical experiments. The last section concludes.

9 Model

9.1 Basic Environment

Time is discrete and infinite, $t = 0, 1, 2, \dots$

There are I types of agents indexed by $i \in \mathcal{I}$. Each type is representative of a set of identical agents in a continuum of measure one. (I will conduct most of my analysis under the special assumption that $\#\mathcal{I} = 2$.)

¹²Tellarini (2000) similarly interprets capital as the equity for in a model of asset pricing in a real business cycles model.

Uncertainty is characterized by an exogenous Markovian random variable s_t governed by probability measure π . I assume that s_t takes values in a finite set \mathcal{S} . I write $s^t := (s_0, s_1, \dots, s_t)$ for a (finite) history. I write $\pi(s^t)$ for the probability of history s^t obtaining from the initial state s_0 , taken as given. The probability of history s^τ following from $s^t \leq s^\tau$ will be written as $\pi(s^\tau | s^t)$. (Note that it is no abuse of notation under these assumptions to write $\pi(s' | s_t)$ for $\pi(s^t, s' | s^t)$.)

There is a single unit of a single good available in each period in each state in the economy. (I will show below how the apparatus I construct may be used to study an economy with permanent growth innovations.) The good is useful only for consumption and only within the period of its dating. Writing $c^i(s^t)$ for the consumption of agent i in history s^t , *resource feasibility* in the economy refers to the constraints that, for each history,

$$c^i(s^t) \geq 0 \text{ and } \sum_i c^i(s^t) \leq 1. \quad (27)$$

There is a single unit of each of two perfectly divisible real assets in the economy. Agent i 's ownership of the first (second) asset at the beginning of history s^t is denoted

by $k^i(s^{t-1})$ (respectively, $b^i(s^{t-1})$). A feasible distribution of these assets satisfies

$$k^i(s^{t-1}) \geq 0 \text{ and } \sum_i k^i(s^{t-1}) = 1 \quad (28)$$

for all s^t , and the analogous conditions for $b(s^{t-1})$. The initial distribution, say $(k(s^{-1}), b(s^{-1}))$, is given at history s_0 . Properties unique to each asset are discussed in detail in the next subsection.

The good is assigned to agents at the beginning of each period as follows. First, to each agent i accrues a personal endowment $w^i(s^t)$. Throughout, I will assume that $w^i(s^t) = (1 - \alpha) \varepsilon^i(s_t)$, where $\alpha \in [0, 1]$ and $\varepsilon^i(s_t) > 0$ for each i ; and $\sum_i \varepsilon^i(s_t) = 1$. Although $w^i(s^t)$ has no literally idiosyncratic component in our setting with a finite number of agent types, it is in keeping with the literature to interpret it as analogous to idiosyncratic income.

Second, to the holder i of the portfolio $(k^i(s^{t-1}), b^i(s^{t-1}))$ accrues the dividend

$$y_k(s^t) k^i(s^{t-1}) + y_b(s^t) b^i(s^{t-1}).$$

In this paper, I will restrict attention to the special case that $y_k(s^t) = \theta \alpha$ and $y_b(s^t) = (1 - \theta) \alpha$ for all histories s^t , where $\theta \in [0, 1]$. The parameter α is thus the share of aggregate income that is attached to transferable dividend generating assets.

In the middle ("market") phase of each period, agents may participate in competitive markets for the real assets, k and b ; and for state-contingent ("financial") claims to goods payable in the market phase of the subsequent period. I write $q(s^t)$ (respectively, $r(s^t)$) for the price at s^t of a unit of the first (second) asset, and I write $p(s'|s^t)$ for the price at s^t of a claim to a unit of the good in history (s^t, s') . (The consumption good in the period of contracting serves as the numeraire for each price.) Writing $a^i(s^t, s_{t+1})$ for i 's purchase at s^t of claims to the s^{t+1} good, the *budget constraint* of this agent is

$$\begin{aligned} & c^i(s^t) + q(s^t) k^i(s^t) + r(s^t) b^i(s^t) + \sum_{s' \in \mathcal{S}} p(s'|s^t) a^i(s^t, s') \\ & \leq w^i(s^t) + [q(s^t) + y_k(s^t)] k^i(s^{t-1}) + [r(s^t) + y_b(s^t)] b^i(s^{t-1}) + a^i(s^t). \end{aligned} \quad (29)$$

Agents consume at the end of each period. Each agent acts at s^t in a time consistent manner to maximize the current discounted value of his utility from consumption. I allow subjective discounting of payoff received after the current period to depend on the current state s_t . Precisely, let the discount factor applied in state s_t to payoff received in subsequent periods be $\beta(s_t)$, where $\beta : \mathcal{S} \rightarrow (0, 1)$. It will be convenient to write $\beta_t(s^\tau) := \prod_{T=t}^{\tau-1} \beta(s_T)$; note that

$$\beta_t(s^\tau) = \frac{\beta_0(s^\tau)}{\beta_0(s^t)} \text{ for } \tau \geq t,$$

and, in particular, $\beta_t(s^{t+1}) = \beta(s_t)$.¹³

The s^t -continuation payoff to agent i of consumption process c can now be defined as

$$U^i(c|s^t) := \sum_{s^\tau \geq s^t} \beta_t(s^\tau) u(c^i(s^\tau)) \pi(s^\tau|s^t). \quad (30)$$

I will assume that $u(\cdot)$ is of the constant relative risk aversion form; that is

$$u(\bar{c}) = \frac{\bar{c}^{1-\sigma} - 1}{1 - \sigma}$$

for some $\sigma > 0$, and I interpret $\sigma = 1$ to mean that $u(\bar{c}) = \ln \bar{c}$.

9.2 Enforcement Technology

Obligations of individual agents to deliver goods inherent in their issue (sale) of claims are fulfilled only voluntarily, where default can be punished only as follows. Upon the decision at the beginning of history s^t of an agent i to abrogate contingent claims obligations, his holdings of asset 2 are confiscated, and he is immediately and forever thereafter banned from participating in the financial (contingent claims) markets and the market for asset 2. It is assumed, however, that his holdings of asset 1 and his ability to trade that asset in spot markets cannot be revoked. The agent

¹³The value of this state contingent discounting will be apparent when I interpret the economy as one with aggregate growth.

retains $k^i(s^{t-1})$ and his ability to buy and sell the asset at his will in present and future histories.

Since agents are non-atomic, the decision of a single agent to default on obligations to supply goods to the market has no effect on aggregate resources. Moreover, I will restrict attention to equilibria wherein agents choose not to default on contracts if they have a weak incentive to comply with them; and where the defection of any single agent (or finite number of agents) is ignored in terms of the behavior of the others.

I assume that market participants, acting with perfecting information, prevent the choosing by any agent of a portfolio that would leave him with a (strict) desire to default in some subsequent history. This mechanism, together with my assumptions about the behavior of other agents following a default, implies that the payoff that any agent expects at any continuation must (weakly) dominate that the agent could achieve for himself were he to commit a default.

The *default payoff problem* of an agent i with ϕ^i shares of asset 1 at the beginning of history s^t is

$$V^i(\phi^i|s^t) := \max u(d^i) + \beta(s_t) \sum_{s'} V^i(\hat{\phi}^i|(s^t, s')) \pi(s'|s_t)$$

subject to

$$d^i + q(s^t) \hat{\phi}^i \leq w^i(s_t) + [q(s^t) + y_k(s_t)] \phi^i$$

and a non-negativity constraint $\hat{\phi}^i \geq 0$, where the choice variables are $(d^i, \hat{\phi}^i)$; and ϕ^i and the price system q are taken as given.¹⁴ I will say that the tuple (c, k, b, a, p, q, r) satisfies the *enforcement constraints* if

$$U^i(c|s^t) \geq V^i(k^i(s^{t-1})|s^t) \quad (31)$$

for all $s^t > s_0$ and i .

The *market payoff problem* of agent i at history s^t when i has not previously defaulted can now be defined as that of maximizing (30) subject to the budget constraints (29) for $s^\tau \geq s^t$, the enforcement constraints (31) for $s^{\tau+1} > s^t$, and non-negative asset holdings $k^i(s^\tau) \geq 0$ and $b^i(s^\tau) \geq 0$ for all $s^\tau \geq s^t$. For the agent's problem at s^t , $(c^i(s^T), k^i(s^{T-1}), b^i(s^{T-1}), a^i(s^T))_{T \leq t}$ and the price processes are taken as given, and the agent chooses the continuations of the processes (c^i, k^i, a^i) relevant to his continuation payoff.

I will refer to asset 1 as the "non-collateral" asset, and asset 2 as the "collateral" assets.

¹⁴I leave the dependence of the function V^i on the price system q implicit in my notation.

9.3 Definition of Equilibrium

An *equilibrium* is a tuple of a real allocation (c, k, b) , prices (q, p, r) , and contingent claims holdings a satisfying the following conditions.

1. For each i and for each s^t , the relevant continuation of (c^i, k^i, b^i, a) solves i 's market problem at s^t , given initial conditions $k^i(s^{t-1})$, $b^i(s^{t-1})$, and $a^i(s^t)$.
2. For each s^t , the markets for goods, assets, and contingent claims clear:

$$\sum_i c^i(s^t) = 1;$$

$$\sum_i k^i(s^t) = 1;$$

$$\sum_i b^i(s^t) = 1;$$

and

$$\sum_i a^i(s^t, s') = 0 \text{ for each } s' \in \mathcal{S}.$$

9.4 An Economy with Aggregate Growth

As explained by Alvarez and Jermann (2000,2001), an economy with stochastic aggregate endowment innovations can be transformed into one fitting the description above as follows.

Consider economy (different from the one described in this paper) in which the aggregate endowment at history s^t is $\Psi(s^t) := \prod_{\tau=0}^t \psi(s_\tau)$, where s^t is now assumed to follow $\hat{\pi}$, a probability measure with the properties ascribed to π above. Suppose further that agents discount future utility by some constant discount factor $\hat{\beta} \in (0, 1)$ in each period, so that i 's s^t -continuation payoff from consuming \hat{c} is

$$\sum_{s^\tau \geq s^t} \hat{\beta}^{\tau-t} \frac{[\hat{c}(s^\tau)]^{1-\sigma}}{1-\sigma} \hat{\pi}(s^\tau | s^t).$$

Now by defining $\beta(\cdot)$ and $\pi(\cdot)$ to be consistent with

$$\beta(s) := \hat{\beta} \sum_{s'} [\psi(s')]^{1-\sigma} \hat{\pi}(s' | s)$$

for each s , and

$$\pi(s' | s) := \frac{\hat{\pi}(s' | s) [\psi(s')]^{1-\sigma}}{\sum_{s'} [\psi(s')]^{1-\sigma} \hat{\pi}(s' | s)},$$

for each s and s' , an economy with the properties described previously can be studied with the interpretation that $c^i(s^t)$, $w^i(s^t)$ and $r_j(s^t)$ are *shares* (i.e., fractions) of the aggregate quantity $\Psi(s^t)$ for each s^t . In what follows, I will show how equilibrium prices for my economy may similarly be transformed to coincide with those of an equilibrium of the economy with growth.

10 Analysis

Because of the dependence of the default payoff functions on expectations about future endogenous variables, the equilibrium allocations cannot easily be summarized as solutions of a central planning problem. In this section, I develop a characterization of a class of recursive equilibria based on an analysis of the decentralized problem. In the next section, I show how equilibria may be computed by an algorithm based on first-order conditions.

For the remainder of the paper, I restrict attention to economies with two types of agents, $\mathcal{I} = \{1, 2\}$.

10.1 Recursive Equilibria

I begin by documenting the form of the first-order necessary conditions for the market payoff problem of agent i . Writing $\beta_0(s^t) \eta^i(s^t) \pi(s^t)$ for the multiplier on the s^t enforcement constraint, the Lagrangian function for this problem may be written as the sum of the terms

$$\begin{aligned} & \sum_{s^\tau \geq s_0} \beta_0(s^\tau) u(c^i(s^\tau)) \pi(s^\tau | s^t) \\ & + \sum_{s^t > s_0} \beta_0(s^t) \eta^i(s^t) \pi(s^t) \left[\sum_{s^\tau \geq s^t} \beta_t(s^\tau) u(c^i(s^\tau)) \pi(s^\tau | s^t) - V^i(k^i(s^{t-1}) | s^t) \right] \end{aligned}$$

plus the sum of the budget constraints weighted by another Lagrange multiplier.

Defining the normalized multiplier

$$\tilde{\eta}^i(s^t) := \frac{\eta^i(s^t)}{1 + \sum_{\tau=0}^t \eta^i(s^\tau)},$$

the first-order and complementary slackness conditions can be summarized by

$$u'(c^i(s^t)) p(s'|s^t) = \beta(s_t) u'(c^i(s^t, s')) [1 + \tilde{\eta}^i(s^t, s')] \pi(s'|s^t) \quad \text{for each } s', \quad (32)$$

$$\begin{aligned} & -u'(c^i(s^t)) q(s^t) \\ & + \beta(s_t) \sum_{s'} u'(c^i(s^t, s')) [1 + \tilde{\eta}^i(s^t, s')] [q(s^t, s') + y_k(s')] \pi(s'|s^t) \\ & - \beta(s_t) \sum_{s'} \tilde{\eta}^i(s^t, s') V_\phi^i(k^i(s^t) | s^t, s') \pi(s'|s^t) \\ \leq & 0, \quad = 0 \text{ if } k^i(s^t) > 0, \end{aligned} \quad (33)$$

$$\begin{aligned} & -u'(c^i(s^t)) r(s^t) \\ & + \beta(s_t) \sum_{s'} u'(c^i(s^t, s')) [1 + \tilde{\eta}^i(s^t, s')] [r(s^t, s') + y_b(s')] \pi(s'|s^t) \\ \leq & 0, \quad = 0 \text{ if } b^i(s^t) > 0, \end{aligned} \quad (34)$$

and

$$\sum_{s^\tau \geq s^t} \beta_t(s^\tau) u(c^i(s^\tau)) \pi(s^\tau | s^t) - V^i(k^i(s^{t-1}) | s^t) \geq 0, \quad = 0 \text{ if } \tilde{\eta}^i(s^t) > 0.$$

Observe that (32) and (34) directly imply that

$$r(s^t) \geq \sum_{s'} p(s' | s^t) [r(s^t, s') + y_b(s')]. \quad (35)$$

Suppose that the inequality were strict for some s^t . In this case, it is clear that no agent would hold the collateral asset at s^t , since replacing the portfolio policy (k^i, b^i, a^i) where $b^i(s^t) > 0$ with $(k^i, \tilde{b}^i, \tilde{a}^i)$ satisfying $\tilde{b}^i(s^t) = 0$ and $\tilde{b}^i(\sigma^\tau) = b^i(\sigma^\tau)$ for $\sigma^\tau \neq s^t$; and $\tilde{a}^i(s' | s^t) = a^i(s' | s^t) + [r(s^t, s') + y_b(s')] b^i(s^t)$ for each s' , and $\tilde{a}^i(\sigma^\tau) = a^i(s' | \sigma^\tau)$ for $\sigma^\tau \neq s^t$ is easily seen to afford an improved consumption stream. Thus it is seen that equilibrium must have (35) holding with equality. Moreover, at equality, agents are indifferent about the ownership of the collateral asset. The following proposition summarizes and extends this idea.

Proposition 13 *In any equilibrium (c, k, b, a, p, q, r) ,*

$$r(s^t) = \sum_{s'} p(s' | s^t) [r(s^t, s') + y_b(s')];$$

and for any stochastic process ξ adapted to π with $\xi(s^t) \in [0, 1]$ for each s^t , there exists an equilibrium with $b^1(s^t) = \xi(s^t)$.

This result points to the nature of collateral itself: a claim on an agent who holds collateral is as good as a claim to the collateral itself. It can be interpreted to hold in the standard frictionless setting of Arrow-Debreu economies for any asset that is redundant in the sense of spanning the uncertainty in the economy. It implies that no trade in the collateral asset is ever necessary. While familiar, this feature of the standard paradigm is counterfactual; the volume of trade in real asset markets is robust, and the fact that real world trade is (even moderately) costly belies the notion that such trade is unnecessary for agents to achieve their desired allocations. The more interesting feature of the present model will be seen in the failure of this proposition applied to non-collateral assets.

Next, considering jointly the Euler equations (32) for the two agents, it can be derived that

$$\frac{u'(c^2(s^t))}{u'(c^1(s^t))} = \frac{[1 + \tilde{\eta}^2(s^t, s')]}{[1 + \tilde{\eta}^1(s^t, s')]} \frac{u'(c^2(s^t, s'))}{u'(c^1(s^t, s'))}.$$

Now define $\gamma(s^t)$ by

$$\frac{\gamma(s^t)}{1 - \gamma(s^t)} \equiv \frac{u'(c^2(s^t))}{u'(c^1(s^t))},$$

and note that equilibrium defines a stochastic process γ whose evolution is governed

by the transition relationship

$$\gamma(s^t, s') = \frac{\gamma(s^t) [1 + \tilde{\eta}^1(s^t, s')]}{1 + \gamma(s^t) \tilde{\eta}^1(s^t, s') + (1 - \gamma(s^t)) \tilde{\eta}^2(s^t, s')}. \quad (36)$$

It is clear that the process γ pins down the consumption processes of the two agents (using the binding resource constraint (27)). The following proposition implies that it pins down the multipliers $\tilde{\eta}^1$ and $\tilde{\eta}^2$ as well whenever the dividend paid to collateral assets is positive.

Lemma 14 *Suppose that $\theta < 1$. In any equilibrium (c, k, b, a, p, q, r) , for each history s^t , at most one agent's enforcement constraint may bind; therefore, at most one of the multipliers $\tilde{\eta}^1(s^t)$ and $\tilde{\eta}^2(s^t)$ can be positive.*

Proof. The hypothesis that $\theta < 1$ implies that some agent i has more wealth at s^t when he does not default; that is, for some i ,

$$\begin{aligned} & w^i(s^t) + [q(s^t) + y_k(s^t)] k^i(s^t) + [r(s^t) + y_b(s^t)] b^i(s^t) + a^i(s^t) \\ & > w^i(s^t) + [q(s^t) + y_k(s^t)] k^i(s^t). \end{aligned}$$

(If not, then adding across agents gives

$$(1 - \alpha) + [q(s^t) + \alpha\theta] + [r(s^t) + \alpha(1 - \theta)] \leq (1 - \alpha) + [q(s^t) + \alpha\theta],$$

a contradiction.) But now it is easy to see that this agent can obtain a more preferred consumption stream in the market than in default; in particular, it suffices to note that the constraints of the market problem afford a consumption stream that is strictly greater than that under an optimal default policy at s^t and identical thereafter. Thus, i 's enforcement constraint cannot be binding at s^t . ■

As mentioned above, the lemma implies that $\gamma(s^t)$ and $\gamma(s^t, s')$ uniquely identify the multipliers $\tilde{\eta}^1(s^t)$ and $\tilde{\eta}^2(s^t)$ through the transition relationship (36) whenever $\theta < 1$. From the first-order conditions and Proposition 13 above, it is also clear that γ pins down the prices p and r as well under these circumstances.

Henceforth, I will restrict attention to the case that $\theta < 1$, and I will study an equilibrium that can be characterized recursively as follows.

Define the state of the economy by $x_t = (\gamma(s^{t-1}), k^1(s^{t-1}), s_t) \in X := [0, 1] \times [0, 1] \times \mathcal{S}$, and let $c^i(x_t) \in \mathbb{R}_+$, $\tilde{k}^i(x_t) \in [0, 1]$, $\tilde{\eta}^i(x_t) \in \mathbb{R}_+$, $q(x_t) \in \mathbb{R}_+$, and $\tilde{\gamma}(x_t) \in [0, 1]$ be functions on X . Define a transition function on X by

$$\chi(x_t, s_{t+1}) := \left(\tilde{\gamma}(x_t), \tilde{k}^1(x_t), s_{t+1} \right).$$

Define

$$p(s'|x_t) := \beta(s_t) \frac{u'(c^1(\chi(x_t, s')))}{u'(c^1(x_t))} [1 + \tilde{\eta}^1(\chi(x_t, s'))] \pi(s'|s^t).$$

Define the function $a^i : X \rightarrow \mathbb{R}$ as the solution to the functional equation

$$a^i(x_t) \equiv c^i(x_t) + q(x_t) \tilde{k}^i(x_t) - w^i(s_t) - [q(x_t) + \alpha\theta] k^i + \sum_{s'} p(s'|x_t) a^i(\chi(x_t, s')).$$

Next for $x = (\gamma, k, s) \in X$, define $V^i(\cdot|x)$ by

$$V^i(\phi^i|x) = \max u(d^i) + \beta(s) \sum_s V^i(\hat{\phi}^i|\chi(x, s')) \pi(s'|s)$$

subject to

$$d^i + q(x) \hat{\phi}^i \leq w^i(s) + [q(x) + \alpha\theta] \hat{\phi}^i$$

and $\hat{\phi}^i \geq 0$. Define W^i by

$$W^i(k^i, a^i|x) = \max u(\hat{c}^i) + \beta(s) \sum_{s'} W^i(\hat{k}^i, \hat{a}^i(s')|\chi(x, s')) \pi(s'|s)$$

subject to

$$\hat{c}^i + q(x) \hat{k}^i + \sum_{s'} p(s'|x) \hat{a}(s') \leq w^i(s) + [q(x) + \alpha\theta] k^i + a^i,$$

$$W^i(\hat{k}^i, \hat{a}^i(s')) \geq V^i(\hat{k}^i|x) \text{ for each } s',$$

and $\hat{k}^i \geq 0$, where the maximization is over choice variables $(\hat{c}^i, \hat{k}^i, \hat{a}^i(\cdot))$.

Now I will say that the functions $(c^i, \tilde{k}^i, \tilde{\eta}^i, q, \tilde{\gamma})$ constitute a *recursive equilibrium* if the following conditions hold.

1. For each $x = (\gamma, k, s) \in X$, $(c^i(x), \tilde{k}^i(x), a^i(\chi(x, \cdot)))$ solves the program defined by $W^i(k^i, a^i(x) | x)$, where $k^1 = k$ and $k^2 = 1 - k$, and $\tilde{\eta}^i(\chi(x, s'))$ is the Lagrange multiplier on the enforcement constraint for s' in the definition of $W^i(k^i, a(x) | x)$.
2. For each $x \in X$, $\sum_i c^i(x) = 1$.
3. For each $x \in X$, $\sum_i \tilde{k}^i(x) = 1$.
4. For each $x = (\gamma, k, s) \in X$,

$$\tilde{\gamma}(x) = \frac{\gamma [1 + \tilde{\eta}^1(x)]}{1 + \gamma \tilde{\eta}^1(x) + (1 - \gamma) \tilde{\eta}^2(x)}.$$

10.2 Asset Pricing

To gain an intuition for the mechanism that gives rise to equilibrium prices in the present model, and to understand how that mechanism is distinct from alternative models, it is useful to begin by reviewing the pricing of a generic redundant asset in the canonical environment of frictionless complete markets. In this context, a representative agent may be thought to choose consumption and an asset portfolio.

The key assumption (retained here) is that, among other asset available for purchase or issue, a complete set of Arrow-Debreu state-contingent securities is offered. The agent maximizes utility subject to a budget constraint like

$$c^i(s^t) + \sum_{s'} p(s'|s^t) a^i(s^t, s') + \sum_j b_j^i(s^t) \leq w^i(s_t) + a^i(s^t) + \sum_j R_j(s^t) b_j^i(s^{t-1}),$$

where $b_j^i(s^t)$ accounts for i 's purchase of (redundant) asset j in history s^t , and the (possibly random) return of that asset is $R_j(s^t, s')$ in history (s^t, s') . (Here, I have considered for simplicity assets that live for only one period and then expire.) In the frictionless paradigm (without enforcement constraints), optimization gives rise to the Euler relationship relevant to asset j

$$1 = \sum_{s'} m(s^t, s') R_j(s^t, s') \pi(s'|s),$$

where the "asset pricing kernel" is

$$m(s^t, s') = m^i(s^t, s') := \beta(s) \frac{u'(c^i(s^t, s'))}{u'(c^i(s^t))}.$$

(Equilibrium and the Euler equation for claims to goods in history (s^t, s') imply that m is independent of i .) If asset f is "risk free" so that $R_f(s^t, s') = \bar{R}_f(s^t)$ for all s' ,

then we have that

$$\bar{R}_f(s^t) = \frac{1}{\sum_{s'} m(s^t, s') \pi(s'|s)}.$$

For any asset j , the Euler equation above can be written as

$$\begin{aligned} 1 &= \left\{ \sum_{s'} m(s^t, s') \pi(s'|s) \right\} \left\{ \sum_{s'} R_j(s^t, s') \pi(s'|s) \right\} \\ &\quad + \sum_{s'} \left\{ m(s^t, s') - \sum_{\sigma} m(s^t, \sigma) \pi(\sigma|s) \right\} \\ &\quad \times \left\{ R_j(s^t, s') - \sum_{\sigma} R_j(s^t, \sigma) \pi(\sigma|s) \right\} \pi(s'|s); \end{aligned}$$

Here, the right-hand side is the product of the conditional expectations of the pricing kernel and the rate of return on asset j , plus the conditional covariance of the two.

This gives rise to

$$\begin{aligned} &\sum_{s'} R_j(s^t, s') \pi(s'|s) - \bar{R}_f(s^t) \\ &= -\bar{R}_f(s^t) \sum_{s'} \left\{ m(s^t, s') - \sum_{\sigma} m(s^t, \sigma) \pi(\sigma|s) \right\} \\ &\quad \times \left\{ R_j(s^t, s') - \sum_{\sigma} R_j(s^t, \sigma) \pi(\sigma|s) \right\} \pi(s'|s). \end{aligned}$$

This is a familiar representation of the excess return attached to asset j over the rate of return on the risk-free asset, and the familiar interpretation is that asset j will

earn a positive "risk premium" if its return covaries negatively with the marginal utility growth of the representative agent – i.e., positively with the growth of his consumption. This explanation is incorporated as a piece of the explanation for the observed premium return of the market portfolio over (essentially) riskless bonds. But the numerical value of this explanation turns out to be tiny if agents preferences are of the standard form under an assumption about the degree of risk aversion of agents that is plausible. This is the "high equity premium puzzle".

On the other hand, if one is willing to entertain a degree of risk aversion high enough to account for the market risk premium in this model, then an alternative puzzle appears. High degrees of risk aversion give rise to a risk free rate $\bar{R}_f(s^t)$ that is inconsistent with low historical observations. This phenomenon is sometimes thought of independently as the "low risk-free rate puzzle".

In an environment with commitment constraints (like the one considered here), the analysis above must be augmented by accounting for the possibility that agents' portfolio choices may be constrained. In this case, the *asset pricing kernel* applicable to collateralizable assets is

$$\tilde{m}(s^t, s') = m^i(s^t, s') + \nu^i(s^t, s')$$

where

$$\nu^i(s^t, s') := \beta(s) \tilde{\eta}^i(s^t, s') \frac{u'(c^i(s^t, s'))}{u'(c^i(s^t))},$$

and $\tilde{\eta}^i(s^t, s')$ represents a non-negative Lagrange multiplier associated with the enforcement constraint applying to history (s^t, s') .

The expression for the risk premium attached to asset j is now

$$\begin{aligned} & -\bar{R}_f(s^t) \sum_{s'} \left\{ m^i(s^t, s') - \sum_{\sigma} m^i(s^t, \sigma) \pi(\sigma|s) \right\} \\ & \times \left\{ R_j(s^t, s') - \sum_{\sigma} R_j(s^t, \sigma) \pi(\sigma|s) \right\} \pi(s'|s) \\ & -\bar{R}_f(s^t) \sum_{s'} \left\{ \nu^i(s^t, s') - \sum_{\sigma} \nu^i(s^t, \sigma) \pi(\sigma|s) \right\} \\ & \times \left\{ R_j(s^t, s') - \sum_{\sigma} R_j(s^t, \sigma) \pi(\sigma|s) \right\} \pi(s'|s). \end{aligned}$$

Thus, the stochastic relationship between borrowing constraints and asset returns enters into the mix. Now it can be seen that negative covariance between the term $\nu^i(s^t, s')$ and the asset return exerts a positive effect on the risk premium associated with the asset.

Alvarez and Jermann (2001) show how this effect plays out numerically. In their calibration, the sign as well as the magnitude of the risk premium depend critically on whether or not the conditional distribution of private endowments is more volatile

in recessions. When the standard deviation of ε^i is assumed to increase in recessions, they find a positive risk premium; when the reverse is assumed to occur, the risk premium is negative.

For the non-collateral asset, the present environment exhibits an additional term in the expression for the risk premium; this term can be written as

$$+\bar{R}_f(s^t) \beta(s_t) \sum_{s'} \frac{\tilde{\eta}^i(s^t, s')}{u'(c^i(s^t)) q(s^t)} V_\phi^i(k^i(s^t) | s^t, s') \pi(s' | s^t).$$

From the envelope theorem,

$$V_\phi^i(k^i(s^t) | s^t, s') = u'(d^i(s^t, s')) [q(s^t, s') + y_k(s')],$$

where d^i represents the optimal consumption of the defecting agent, so that the term is equal to

$$+\bar{R}_f(s^t) \sum_{s'} \delta^i(s^t, s') R_k(s^t, s') \pi(s' | s^t),$$

where

$$\delta^i(s^t, s') := \beta(s) \tilde{\eta}^i(s^t, s') \frac{u'(d^i(s^t, s'))}{u'(c^i(s^t))}.$$

A stochastic discount factor applicable to non-collateral assets is

$$\hat{m}^i(s^t, s') = m^i(s^t, s') + \nu^i(s^t, s') - \delta^i(s^t, s').$$

The new term $\delta^i(s^t, s')$ has a structure like that of $v^i(s^t, s')$, except that agent i 's intertemporal marginal rate of substitution in the definition of the latter is replaced by the intertemporal marginal rate of substitution that would apply were he to default in state (s^t, s') . This term embodies a constraint faced by agent i by expanded opportunity in default as a result of holding a larger quantity of the non-collateral asset. Intuitively, if the agent increases his holdings at s^t , he increases his default payoff in state (s^t, s') . In the states (s^t, s') for which his enforcement constraint binds, the increase in default payoff reduces the amount of debt that he can credibly promise to repay. Although it is a complex matter to anticipate the effect of the new term in the pricing kernel for the implied return premium on non-collateral assets, it is unambiguously positive. In the next section, I study the quantitative implications of the model, and I compare it to other results in the literature by interpreting the non-collateral asset as "equity".

11 Numerical Experiments

11.1 Computation Strategy

The algorithm I use to compute a recursive equilibrium of the economy is a significant extension of that used by Kehoe and Perri (2002).

The algorithm is an iterative one based on improvement from an initial starting guess. For the latter, I begin by computing, for each $x = (\gamma, k, s)$, the allocation $\left(c_0^i(x), \tilde{k}_0^i(x), \tilde{\eta}_0^i(x), q_0(x), \tilde{\gamma}_0(x)\right)$ and the transition rule $\chi_0(x, \cdot)$ that would apply if the enforcement constraints did not need to be imposed – i.e., the first-best allocation. I let $\tilde{W}_0^i(x)$ be the payoff to i under this allocation defined by the solution to

$$\tilde{W}_0^i(x) = u(c_0^i(x)) + \beta(s) \sum_{s'} \tilde{W}_0^i(\chi_0(x, s')) \pi(s'|s).$$

For the defection payoff, one can use standard recursive procedures to compute the payoff for each $x = (\gamma, k, s)$ of having capital ϕ in the incomplete markets setting under the first-best price system q_0 , when transitions on the aggregate state abide by the transition rule χ_0 . However, it is necessary to take care that the enforcement constraints can be satisfied by some feasible allocation for each x . As a matter of practice, I have found it sufficient to consider starting values for $V_0^i(\phi|x)$ equal to the function just described minus some fixed positive quantity. I have found it

useful also to keep track of the solutions to the problem defined by $V_0^i(\phi|x)$, say $(\hat{d}_0^i(\phi|x), \hat{\phi}_0^i(\phi|x))$. In solving the equations that follow, the need to approximate the derivative $V_{0\phi}^i$ of an approximated function is alleviated by using the envelope theorem result

$$V_{0\phi}^i(\phi|x) = u'(\hat{d}_0^i(\phi|x)) [q_0(x) + y_k(s)].$$

For conceptual clarity, I have retained the $V_{0\phi}^i$ notation below.

For each $x = (\gamma, k, s) \in X = [0, 1] \times [0, 1] \times \mathcal{S}$, I first compute provisional values for the next iterate $(c_1^i(x), \tilde{k}_1^i(x), \tilde{\eta}_1^i(x), q_1(x), \tilde{\gamma}_1(x))$ by assuming that $\tilde{\eta}_1^i(x) = 0$ for each i , and $\tilde{\gamma}_1(x) = \gamma$; i.e., as if neither of the agents' enforcement constraints were binding for state x . These values satisfy the following conditions:

$$\tilde{\gamma}_1(x) u'(c_1^1(x)) = [1 - \tilde{\gamma}_1(x)] u'(c_1^2(x)); \quad (37)$$

$$\begin{aligned} & -u'(c_1^i(x)) q_1(x) \\ & + \beta(s) \sum_{s'} u'(c_1^i(\chi_1(x, s')) [1 + \tilde{\eta}_0^i(\chi_1(x, s'))] [q_0(\chi_1(x, s')) + y_k(s')] \pi(s'|s) \\ & - \beta(s) \sum_{s'} \tilde{\eta}_0^i(\chi_1(x, s')) \tilde{V}_{0\phi}^i(\tilde{k}_1^i(x) | \chi_1(x, s')) \pi(s'|s) \\ & \leq 0, \quad = 0 \text{ if } \tilde{k}_1^i(x) > 0 \end{aligned} \quad (38)$$

for each i ;

$$\sum_i c_1^i(x) = 1; \quad (39)$$

and

$$\sum_i \tilde{k}_1^i(x) = 1; \quad (40)$$

where $\chi_1(x, s') = (\gamma, k_1^1(x), s')$.

Next, I check the enforcement constraints for this choice. The payoff to i of honoring his obligations under the provisional allocation is

$$u(c_1^i(x)) + \beta(s) \sum_{s'} \tilde{W}_0^i\left(\left(\gamma, \tilde{k}_1^1(x), s'\right)\right) \pi(s'|s).$$

In Kehoe and Perri, the default payoff of agent i is an exogenous function of $\tilde{k}_1^i(x)$; in the present environment, however, the default payoff is co-determined with the price system, so that it cannot be calculated independently. While the form of the function $V_1^i(\cdot|x)$ is unknown a priori, provisional choices for consumption and savings of an agent who defaults at x , say $d_1^i(x)$ and $\phi_1^i(x)$ can be derived. In state x , the distribution of the non-collateral asset is described by $k^1 = k$ and $k^2 = 1 - k$; and the proposed allocation suggests that the price of the non-collateral asset follows $q_1(x)$ in the current period and $q_0(\cdot)$ in the subsequent one. In this case, $d_1^i(x)$ and

$\phi_1^i(x)$ must satisfy

$$d_1^i(x) + q_1(x) \phi_1^i(x) = w^i(s) + [q_1(x) + y_k(s)] k^i \quad (41)$$

and

$$-u'(d_1^i(x)) q_1(x) + \beta(s) \sum_{s'} V_{0k}^i(\phi_1^i(x) | \chi_1(x, s')) \pi(s'|s) \leq 0, \quad = 0 \text{ if } \phi_1^i(x) > 0. \quad (42)$$

After solving (41) and (42) for each i , I test the veracity of the inequalities

$$\begin{aligned} & u(c_1^i(x)) + \beta(s) \sum_{s'} \tilde{W}_0^i(\chi_1(x, s')) \pi(s'|s) \\ & \geq u(d_1^i(x)) + \beta(s) \sum_{s'} V_0^i(\phi_1^i(x) | \chi_1(x, s')) \pi(s'|s). \end{aligned} \quad (43)$$

If these are satisfied for each i , I define $\tilde{W}_1^i(x)$ by the left-hand side for each i , and I record the provisional values as the new starting allocation for this state x . If the constraint is satisfied for agent 1 (for example) but not for agent 2,¹⁵ then I set $\tilde{\eta}_1^1(x) = 0$, and I solve for the values $(c_1^i(x), \tilde{k}_1^i(x), q_1(x), \tilde{\gamma}_1(x))$ and $(\tilde{\eta}_1^2(x), d_1^2(x), \phi_1^2(x))$

¹⁵Recall that I have assumed that only one of the constraints binds at a time in the equilibrium considered.

that satisfy (37)-(40), (41)-(42) for $i = 2$, (43) with equality for $i = 2$, and

$$\tilde{\gamma}_1(x) = \frac{\gamma}{1 + (1 - \gamma) \tilde{\eta}_1^2(x)}; \quad (44)$$

where I now take $\chi_1(x, s') = (\tilde{\gamma}_1(x), \tilde{k}_1^1(x), s')$. The solution defines a new allocation for the next iteration for this x . I take $\tilde{W}_1^i(x)$ by evaluating the Left-hand side of (43) for each i under this allocation.

If I had found instead that the constraint is violated for agent 1, I would perform the analogous calculations by switching the appropriate indices, except that (44) is replaced (c.f., equation (36)) by

$$\tilde{\gamma}_1(x) = \frac{\gamma [1 + \tilde{\eta}_1^1(x)]}{1 + \gamma \tilde{\eta}_1^1(x)};$$

Finally, I compute the updated default payoff functions $V_1^i(\cdot|x)$ by finding for each ϕ in an appropriately chosen subset of \mathbb{R}_+ the solution $(\hat{d}_1^i(\phi|x), \hat{\phi}_1^i(\phi|x))$ to the program

$$V_1^i(\phi^i|x) = \max u(\hat{d}_1^i) + \beta(s) \sum_{s'} V_0^i(\hat{\phi}_1^i|\chi_1(x, s'))$$

subject to

$$\hat{d}_1^i + q_1(x) \hat{\phi}_1^i = w^i(s) + [q_1(x) + y_k(s)] \phi^i$$

and $\hat{\phi}_1^i \geq 0$.

After performing these calculations for each x , I have obtained a complete set of updated functions $(c_1^i, \tilde{k}_1^i, \tilde{\eta}_1^i, q_1, \tilde{\gamma}_1)$, a transition rule χ_1 , value functions \tilde{W}_1^i and V_1^i , and defection policy functions $(\hat{d}_1^i, \hat{\phi}_1^i)$. I then compare them to the analogous starting values to assess convergence. If a satisfactory degree of convergence has been achieved the updates represent my approximation of the solution; otherwise, I take the new values in the role of the starting values and repeat the iteration.

For the numerical implementation of the algorithm, I take a finite grid on the state space, and I use tensor product splines to approximate the functions. Because (presumably) of the presence of constraints that bind only occasionally, I have found that using splines that are linear in the dimensions of γ and k work best.¹⁶ Because of the strict concavity of V^i in ϕ , I have obtained good results by allowing this function to be quadratic in this dimension.

¹⁶K-P also use linear splines to compute equilibria for their economy.

11.2 Calibration

For comparison with previous results on asset pricing in economies with limited enforcement, I adopt a calibration strategy in the framework of Alvarez and Jermann (2001). This framework consists in matching ten moments in the data, identified as *M1-M10*.

The exogenous state space consists of four elements, $\mathcal{S} = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$. I assume that the aggregate endowment grows at rate $\psi(s)$ in state s , where

$$\psi(\sigma_1) = \psi(\sigma_2) = \psi_e, \text{ and } \psi(\sigma_3) = \psi(\sigma_4) = \psi_r,$$

with $\psi_e > \psi_r$. States σ_1 and σ_2 are expansions, and states σ_3 and σ_4 are recessions.

Individual income shares are

$$\varepsilon^1(\sigma_1) = 1 - \varepsilon^1(\sigma_2) = \varepsilon_{eh}, \text{ and } \varepsilon^1(\sigma_3) = 1 - \varepsilon^1(\sigma_4) = \varepsilon_{rh},$$

with $\varepsilon_{ah} \in [\frac{1}{2}, 1)$ for $a = e, r$; and $\varepsilon^2 = 1 - \varepsilon^1$. Agent 1 receives a high share of the aggregate income in states 1 and 3; agent 2 receives the high share in states 2 and 4.

The Markov transition matrix is assumed to treat agents symmetrically, so that the matrix can be characterized by six parameters: the probability of switch from expansion to recession, the probability of switch from recession back to expansion,

and a probability of switching from low to high income conditional on the current and the prior aggregate state.

The properties of the aggregate endowment process are pinned down by four moments, $M1-M4$, which I hold fixed throughout the analysis. These pin down the function ψ and the first two parameters of the stochastic matrix.

M1: The first -order serial correlation of $\psi(s)$ is -0.14 . This is in accord with Mehra and Prescott (1985).

M2: The ratio of the unconditional probability of expansion to that of recession is 2.65 . This moment is introduced by Alvarez and Jermann, and reflects data from the NBER for 1889-1991.

M3: The expectation of ψ is 1.0183 (Mehra and Prescott (1985)).

M4: The standard deviation of ψ is 0.0357 (3.57%) (Mehra and Prescott (1985)).

The remaining moments describe the individual income shares. To facilitate comparison to existing studies, and reflecting the fact that the uncertainty inherent in idiosyncratic income is less well understood, I will compute the model under an array of alternative assumptions about the stochastic nature of the income shares. The moments to be matched by the process are the following ones.

M5: The unconditional standard deviation of the log of the income share, σ_ϵ . This value is of first-order importance in calibration of limited enforcement economies. In

the present setting, a high value makes access to financial markets more valuable, because it allows agents to tailor their insurance contracts to smooth this volatility in their income. Similarly, a high value of this moment can make first-best risk sharing enforceable.

M6: The first-order serial correlation of the log of the income share, ρ_ε . High serial correlation in the time-series of an agent's income share reduces the amount of risk-sharing accomplished by pure finance in my model. This is because, in the ergodic distribution in an equilibrium, the agent with the high income tends to be making transfers to the low income agent. If high income today implies a high probability of high income in the future, defaulting on these transfer obligations tends to look pretty attractive, even when it comes at the expense of foregoing insurance against the (small) possibility of reduced future income.

M7: The ratio of the cross-sectional dispersion of the shares in recessions v_r to that in expansions v_e , where the dispersion is defined by

$$v^2(s) = \frac{1}{2} \sum_i \left[\varepsilon^i(s) - \frac{1}{2} \right]^2.$$

(It is worth noting that setting $v_r/v_e = 1$, as is done in the literature following Alvarez and Jermann (2001), is equivalent in this environment to setting $\varepsilon_{eh} = \varepsilon_{rh}$; this relationship holds in each of the parameterizations I study.)

M8: Ratio of standard deviations of income shares in recession to that in expansion conditional on expansion in the previous period.¹⁷

$$\frac{\sigma_{er}}{\sigma_{ee}} = \frac{Std(\ln \varepsilon^1(s^{t+1}) | \psi(s^{t+1}) = \psi_r, \psi(s^t) = \psi_e)}{Std(\ln \varepsilon^1(s^{t+1}) | \psi(s^{t+1}) = \psi_e, \psi(s^t) = \psi_e)}.$$

The important aspect of *M8* and *M9* is that they determine the volatility of individual endowment income in expansions versus recessions. This feature is of first-order importance in theory surrounding the equity premium (as discussed above). Important empirical studies have verified that recessions offer more volatility, an important ingredient in explaining the risk premium.¹⁸ This feature seems sufficient in extant models for the sign of the risk premium. A task before the researchers is to build models that explain the magnitude of that effect.

M9: Ratio of standard deviations of income shares in recession to that in expansion conditional on recession in the previous period:

$$\frac{\sigma_{rr}}{\sigma_{re}} = \frac{Std(\ln \varepsilon^1(s^{t+1}) | \psi(s^{t+1}) = \psi_r, \psi(s^t) = \psi_r)}{Std(\ln \varepsilon^1(s^{t+1}) | \psi(s^{t+1}) = \psi_e, \psi(s^t) = \psi_r)}.$$

M10: The ratio of the standard deviation of the income shares conditional on

¹⁷For *M8* and *M9*, A-J use the notation $\sigma_{r'e}$ to describe what I have defined as σ_{er} .

¹⁸For example, see Heaton and Lucas (1996) and Storesletten, Telmer and Yaron (2004).

prior aggregate shock:

$$\frac{\sigma_r}{\sigma_e} = \frac{Std(\ln \varepsilon^1(s^{t+1}) | \psi(s^t) = \psi_r)}{Std(\ln \varepsilon^1(s^{t+1}) | \psi(s^t) = \psi_r)}.$$

Alvarez and Jermann (2001) discuss the effect of this moment for explaining the term premium. I have not been concerned with this feature of the model.

The remaining parameters of the model calibration determine agents' preferences, the share of assets in income, and the share of collateral in assets. Preference parameters are chosen primarily to accord with convention, and varied to assess the sensitivity of the model to specification changes. Throughout, the momentary utility function is held to be logarithmic, and the discount factor is at least 0.95.

Treatments are chosen to investigate three primary effects. First, I vary the volatility of the endowment process. Second, I assess the importance of the share of collateral in assets. Third, I vary the discount factor within a conventional range to see how well the model can match the moments of interest.

Parameters of the Idiosyncratic Income Process:

<i>M5</i>	(See below)	<i>M8</i>	1.88
<i>M6</i>	0.75	<i>M9</i>	1.88
<i>M7</i>	1	<i>M10</i>	0.95

11.3 Data

The following table summarizes Mehra and Prescott's (1985) data on the equity premium and the risk-free rate for the U.S. for the period 1889-1978. The implied value of Sharpe's ratio is 0.37.

Mehra and Prescott (1985) observed that the conventional model of macroeconomic equilibrium with complete markets is unable to approximate this data in simulations for parameters that are economically reasonable. Comparing this data with that from the simulations of "first-best" economies in the tables below is suggestive of their conclusions.

Statistic	Average	Std Deviation
Return on Equity	6.98%	16.54
Return on Essentially Risk-Free securities	0.80%	5.67
Equity Premium	6.18%	16.67

11.4 Results

Figures 1 and 2 below show the computed transition rules $\tilde{\gamma}(\cdot, k, s)$ and $\tilde{k}^1(\cdot, k, s)$ for different values of k . The flat parts of each plot indicate the regions where the enforcement constraints bind. The lower (γ) plateau of each manifold has agent 1's

constraint binding; the upper one (higher γ) is where agent 2's constraint binds. In the region between, the constraints do not bind; for states (γ, k, s) in this region, $\tilde{\gamma}(\gamma, k, s) = \gamma$.

Figures 3 and 4 plot a simulated time-series. The first shows the state variables (where $s \in \{1, 2, 3, 4\}$ is normalized by $1 + 0.025s$) for 100 periods. Here it can be seen how agent 1's wealth (proxied by the endogenous state variables, γ and k) grows in states in which his endowment is high ($s \in \{1, 3\}$), and falls when it is low. This model is distinguished from the enforcement constraints model with perfect exclusion from assets markets by the gradual adjustment of the weighting γ . The intuition for the slower adjustment is that possession of non-collateral capital acts as a brake (a *real* impediment, as opposed to merely a financial one) on the speed of adjustment of an agent's wealth. This impedes risk sharing, because the wealthy agent has the physical ability to attenuate changes in his wealth.

Intuitively, it can be thought that an agent with significant non-collateral asset holdings would like to sell more of this asset than he is able to when his income falls. Unfortunately, the agent to whom he would sell the asset can only purchase larger quantities of the asset by leveraging future income. But holding more of the non-collateral asset while taking on more debt creates conflicting incentives, inhibiting the arrangement.

The volatility of consumption and non-collateral asset holdings is apparent in the simulated time-series. It is a virtue of the model that the distribution of the non-collateral asset matters in terms of the amount of risk sharing that can be achieved.

The volatility of the idiosyncratic component of the income process has a non-monotone effect on the moments of interest. While higher idiosyncratic volatility in income would be expected to carry through to consumption, it makes state-contingent insurance more valuable and self-insurance less valuable. The net effect is non-monotone for the non-collateral asset premium. If the volatility is pushed high enough, perfect risk sharing may be obtained; this effect essentially works by reducing the payoff to default. While insurance may be essentially unsustainable when volatility is especially low, it also becomes unnecessary as idiosyncratic volatility goes to zero.

I take $\sigma_\varepsilon = 0.40$ as my benchmark for the volatility of the endowment share.¹⁹ Table 1 shows a comparison for of the benchmark to a calibrated with $\sigma_\varepsilon = 0.296$ (as in the benchmark of Alvarez and Jermann (2001)). The increase in volatility carries through to most of the features of the simulation: consumption, non-collateral asset holdings, and the standard deviations of the risk-free rate and the return on the non-collateral asset. It also reduces the risk-free rate and increases the equity premium.

¹⁹The value is based on the study of Heaton and Lucas. It is also the benchmark taken by Lustig (2001).

The share of collateral in assets is the single most sensitive parameter in the calibration, as Table 2 shows. For the parametric specifications considered, for example, perfect risk-sharing obtains for this share greater than ten percent ($\theta \leq 0.9$). This feature of the model can be seen as analogous to other results in the literature. In models with incomplete markets, numerous authors (e.g., Telmer (1993) and Lucas (1994)) have shown that, in calibrated general equilibrium economies, agents are able to shed a great deal of income risk given moderate debt limits. Lucas (1994) documents that a similar effect is observed in an economy in which 30% of income accrues to holders of a tradable dividend-yielding outside asset (stock shares). In an economy with a complete set of state-contingent claims, Lustig (2001) results how a moderate amount of collateral greatly facilitates risk-sharing in a limited enforcement economy.

Increasing the subjective discount factor enhances the value of maintaining a reputation, while it tends also to increase the price of assets yielding dividends in perpetuity. Table 3 documents the result for the moments under study. The latter effect reduces the rate of return on the non-collateral asset, reducing the value of the default payoff. In these experiments, the risk-free rate tracks the reciprocal of the subjective discount factor closely. Therefore, it seems that, for a fixed value of the risk-aversion parameter, β indexes a trade-off between achievement of a high equity

premium and a low risk-free rate.

In parameterizing the model, I have resisted departures from the most conventional preference parameters in order to set the bar high. In particular, the experiments reported below maintain the share of assets in income at $\alpha = 0.33$, the coefficient of relative risk aversion at $\sigma = 1$, and the volatility of agents' endowment shares at moderate levels. I believe that the model shows some important successes, and that this will become more clear as experiments proceed. Under the parametric constraints I have held so far, however, I have not been able to match simultaneously and accurately the risk-free rate and the equity premium. It seems apparent that this cannot be achieved for this low value of the coefficient of relative risk aversion.

At the time of writing, I am conducting experiments with higher degrees of risk aversion and higher values of the discount factor. My casual experiments have led me to be confident that values of σ in the range of 2-3 will easily deliver the high equity premium; values in this range also imply a satisfying level of volatility in the risk-free rate. The net interactive effect of changes in these two parameters is not yet clear to me. Nevertheless, I am confident that the best evidence in favor of the model has yet to be uncovered.

12 Conclusions

This paper introduces a framework for the study of limited enforcement economies in which agents who default on their intertemporal promises retain accumulated holdings of a special "non-collateral" asset, and cannot be excluded from freely trading that asset in the future. In equilibrium, agents must be given incentives not to default. The implied equilibration mechanism may induce significant volatility to the price of the non-collateral asset, and to other endogenous prices and quantities in the model. Significantly, the distribution of the non-collateral asset matters.

By interpreting the non-collateral asset as equity, the model offers a component explanation for the high price of equity on average – the equity premium puzzle. Quantitative experiments reveal that the model can generate a significant equity premium with agents that are not too risk averse. Within the constraints of my parameterization, the risk free rate exhibits too little volatility relative to the data. As of this writing, my experiments fall short of a full explanation for the equity premium and the risk-free rate puzzles; but I have confidence that the contribution will prove to be significant, whether or not the puzzles are fully resolved within the framework.

From this work, and from the work of Seppälä (1999), Kehoe and Perri (2002), and Lustig (2001,2003), it is clear that study of economies with alternative treatment

of default in limited enforcement economies is merited. In particular, I believe that careful modeling of the factors that cause this payoff to vary over time will be most fruitful.

In light of the implications of the model for economies with non-collateral assets, and interpreting results from Telmer (1993), Lucas (1994), and Lustig (2003), I believe that a more thorough empirical accounting of the stock and quality of collateral verses non-collateral assets is warranted. Although research has sought to understand the role of collateral in real and model economies, it seems remiss to fail to understand the behavior of an asset that is *not* collateral. I hope that I have motivated research interest in achieving such an understanding.

13 Figures and Tables for Chapter 2

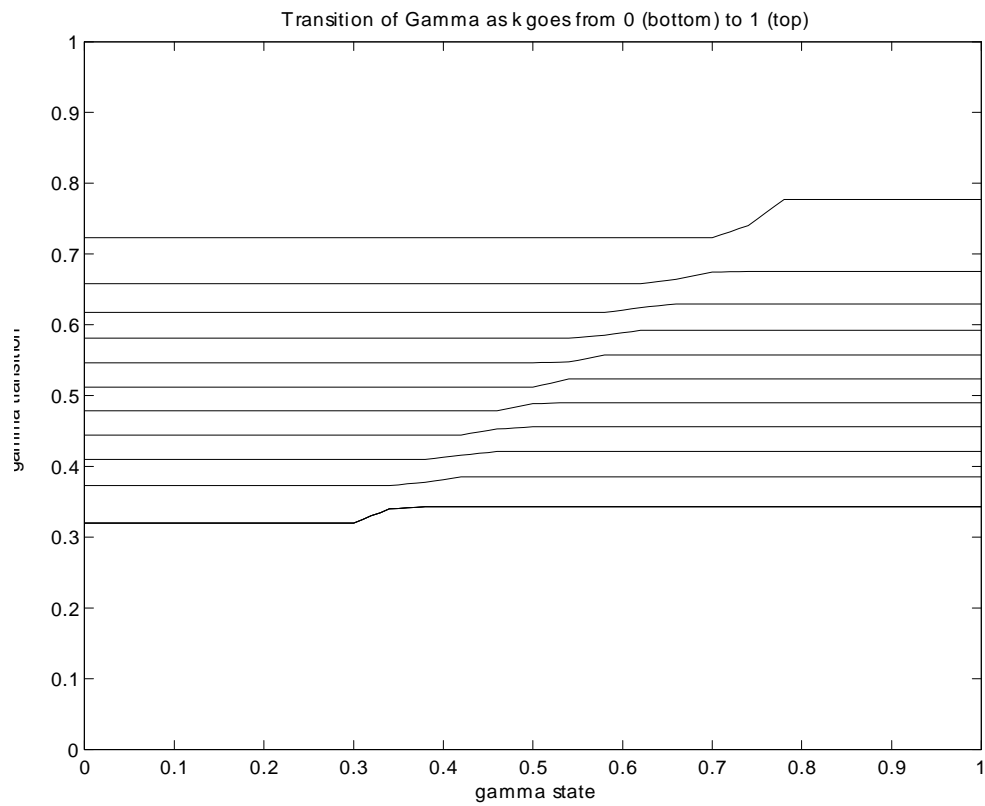


Figure 4. *Transition of γ as k varies from 0 (bottom) to 1 (top).*

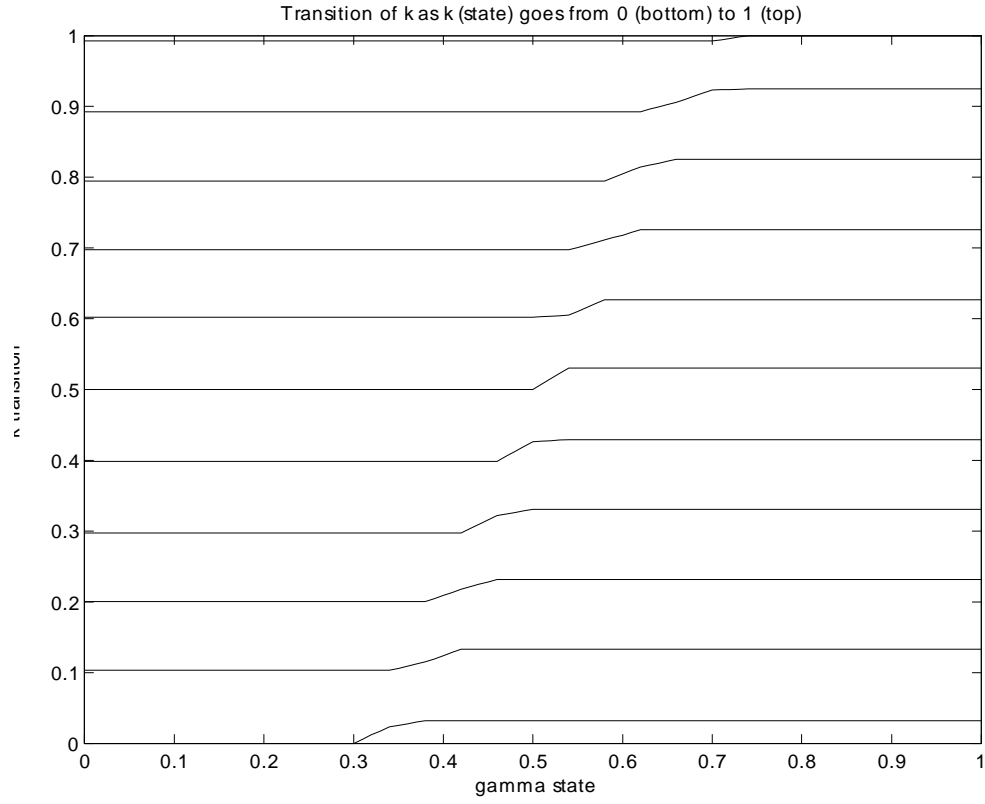


Figure 5. *Transition of k as k (state) varies from 0 (bottom) to 1 (top).*

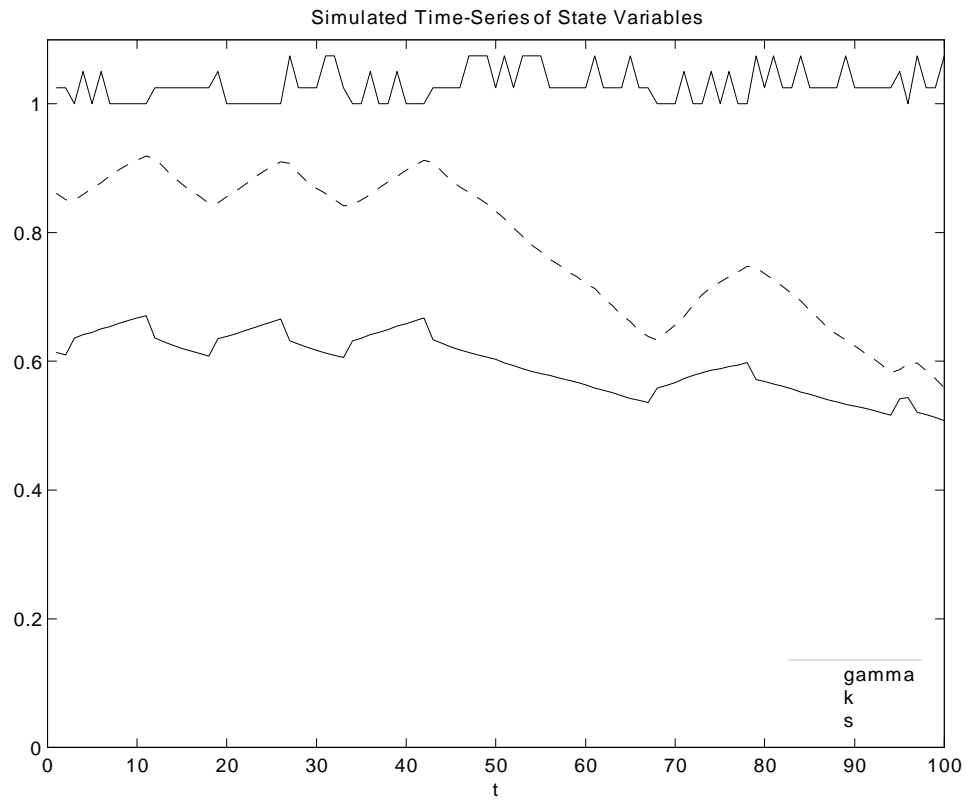


Figure 6. *Simulated Time-Series of State Variables.*

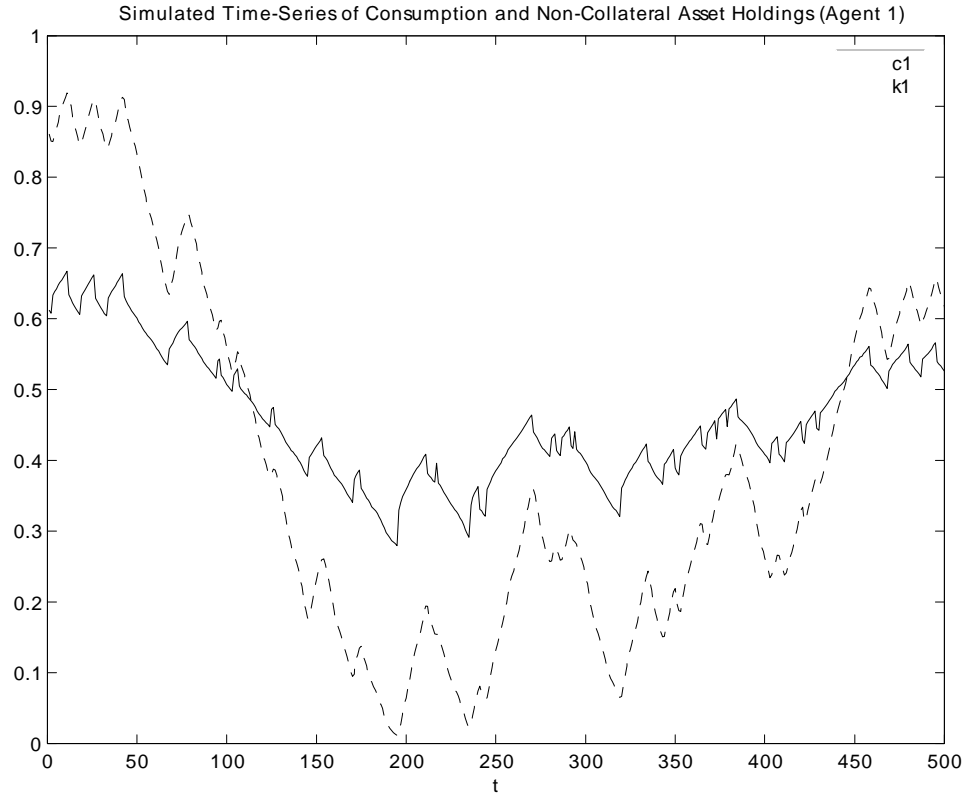


Figure 7. *Simulated Time-Series of Consumption and Non-Collateral Asset Holdings (Agent 1).*

Table 1. *Effect of volatility of endowment income share.*

	$\sigma(c^1)$	$\sigma(k^1)$	$E(R_f)$	$\sigma(R_f)$	$E(R_k)$	$\sigma(R_k)$	$E(R_k - R_f)$
$\sigma_\varepsilon = .296$.0944	.2579	1.0553	.0074	1.0710	.0381	.0157
$\sigma_\varepsilon = .40$.1124	.2805	1.0483	.0086	1.0707	.0389	.0224
First-best	0	0	1.0705	.0055	1.0722	.0374	.0017
US, 1889-1978			1.0080	.057	1.0698	.1667	.0618
All data is preliminary. $\alpha = .33$, $\sigma = 1$, $\beta = 0.95$, $\theta = 0.98$, $\rho_\varepsilon = 0.75$.							

Table 2. *Effect of share of collateral in assets.*

	$\sigma(c^1)$	$\sigma(k^1)$	$E(R_f)$	$\sigma(R_f)$	$E(R_k)$	$\sigma(R_k)$	$E(R_k - R_f)$
$\sigma_\varepsilon = .296$							
$\theta = .98$	0.0944	0.2579	1.0553	.0074	1.0710	.0381	.0157
$\theta = .99$	0.0898	0.2304	1.0528	.0075	1.0715	.0379	.0187
$\sigma_\varepsilon = .40$							
$\theta = .98$.1124	.2805	1.0483	.0086	1.0707	.0389	.0224
$\theta = .99$.1124	.2671	1.0468	.0087	1.0711	.0386	.0243

All data is preliminary. $\alpha = .33$, $\sigma = 1$, $\beta = 0.95$, $\sigma_\varepsilon = 0.40$, $\rho_\varepsilon = 0.75$.

Table 3. *Effect of subjective discount factor, β .*

	$\sigma(c^1)$	$\sigma(k^1)$	$E(R_f)$	$\sigma(R_f)$	$E(R_k)$	$\sigma(R_k)$	$E(R_k - R_f)$
$\beta = 0.95$.1124	.2805	1.0483	.0086	1.0707	.0389	.0224
$\beta = 0.975$.0740	.2049	1.0355	.0062	1.0440	.0369	.0085

All data is preliminary. $\alpha = 0.33$, $\sigma = 1$, $\theta = 0.98$, $\sigma_\varepsilon = 0.40$, $\rho_\varepsilon = 0.75$.

Table 4. *Effect of persistence of endowment income share.*

	$\sigma(c^1)$	$\sigma(k^1)$	$E(R_f)$	$\sigma(R_f)$	$E(R_k)$	$\sigma(R_k)$	$E(R_k - R_f)$
$\sigma_\varepsilon = .40$:							
$\rho_\varepsilon = .75$.1124	.2805	1.0483	.0086	1.0707	.0389	.0224
$\rho_\varepsilon = .90$.1471	.2981	1.0483	.0092	1.0703	0.0379	.0220

All data is preliminary. $\alpha = 0.33$, $\sigma = 1$, $\beta = 0.95$, $\theta = 0.98$, $\sigma_\varepsilon = 0.40$.

Chapter III

Sharing Risk Efficiently when Default Punishments may be Suboptimal

14 Introduction

What characteristics of the consumption processes of a cohort of agents over time are consistent with equilibrium in a complete markets economy? The canonical model of frictionless trade in complete markets allows us to reject the hypothesis whenever we can conclude that intertemporal marginal rates of substitution are not equated across agents at each point in time. Indeed, one is not hard pressed to assemble data that contradict this implication of frictionless complete markets.

Recent research has focused on deriving the observable implications for equilibrium in economies encumbered by enforcement (or "commitment") frictions, but that are otherwise frictionless. Of critical importance in such work is the specification of what agents may accomplish after a behavioral deviation from prescribed or contracted actions; that is, specification of punishments. In this respect, Kehoe and Levine (1993) and Kocherlakota (1996) have set the paradigm adopted by the rest of the literature. Following Abreu (1988), These authors each suppose that agents

are treated to the harshest punishment that is available subject to the exogenously specified "autarkic" capabilities of individuals. In each case, this corresponds to some notion of exclusion from participation in some part of economic life.

While the equilibrium concepts in each of these suggest that agents explicitly acknowledge enforcement constraints, Alvarez and Jermann (2000) suggest that they may be embodied in market determined restrictions on agents' debt. Thus, agents in their setting act in markets in each period facing budget set that includes a system of state-contingent "solvency constraints". When the solvency constraints are set appropriately, equilibria of their economy coincide with Kocherlakota's optima.

The philosophy underlying the present work is that markets may organize the front line (the equilibrium path) of economic behavior, but the consequences to a single agent who deviates from the norm of prescribed behavior is taken as exogenously, possibly suboptimally, determined. More precisely, rather than choose punishments optimally, I assume that deviating agents are punished by reversion to some arbitrary subgame perfect continuation equilibrium. The fundamental result of the paper shows that a very wide class of allocations can be rationalized as equilibria in markets with enforcement constraints.

In the next section, I introduce the environment and the underlying game played by its agents. In the third section, I exposit the principal result of the paper. The

fourth section compares my results to those of Alvarez and Jermann (2000), and states a version of the Second Welfare Theorem for my environment. The final sections discuss my results and conclude.

15 Model

15.1 Environment

Time is discrete and infinite, and is indexed by $t = 0, 1, 2, \dots$. There are $I < \infty$ agents in the economy indexed by $i \in \mathcal{I} = \{1, 2, \dots, I\}$.

Stochastic features of the environment are summarized by a random process s_t that evolves according to a probability measure π . Each s_t is an element of a finite set \mathcal{S} . I write $s^t \in \mathcal{S}^{t+1}$ for the history of the exogenous shock process up to date t . the initial state s_0 will be taken as given and left implicit where there can be no confusion. All stochastic processes in this paper are assumed to be adapted to π . For any such process x , I will write $x|s^t$ for the continuation of x after history s^t ; that is, $x|s^t$ is a stochastic process for initial state s_t . (I assume that π is one-period Markovian.)

There is a single (consumption) good in the economy available at each date. The aggregate endowment of the good is one unit. At each history s^t at which the

state is s_t , agent i is endowed with a fraction $e^i(s_t) > 0$ units of the good, where $\sum_i e^i(s_t) = 1$.

A *feasible allocation* is a stochastic process c such that $c(s^t) \in \mathbb{R}_+^I$, and $\sum_i c^i(s^t) \leq 1$.

After any history s^t , agent i evaluates the continuation allocation $c|s^t$ according to the criterion

$$\mathcal{U}^i(c|s^t) := \sum_{\tau=t}^{\infty} \sum_{s^\tau} \beta^{\tau-t} u(c^i(s^\tau)) \pi(s^\tau|s_t),$$

where $u : \mathbb{R}_+ \rightarrow \mathbb{R} \cup \{-\infty\}$ is strictly concave and strictly increasing. I will also assume that $\lim_{c \rightarrow 0} u(c) = u(0)$, which may be equal to $-\infty$.

It will be useful to define the payoff from autarkic consumption,

$$\mathcal{U}_{aut}^i(s_t) := \sum_{\tau=t}^{\infty} \sum_{s^\tau} \beta^{\tau-t} u(e^i(s_\tau)) \pi(s^\tau|s_t).$$

15.2 A Game of Multilateral Transfers

The game defined here is a generalization of that studied by Kocherlakota (1996) to the case of Markov shocks and an arbitrary number of agents.

The set of actions available to i in history s^t is

$$A^i(s_t) = \left\{ a \in \mathbb{R}_+^I : \sum_j a_j \leq e^i(s_t) \right\}.$$

I will write $a(s^t)$ for the profile of actions taken by each agent after state history s^t .

A game history for the period t is a pairing of a state history s^t and a history of actions played for each $s^{t-\tau}$ for $1 \leq \tau \leq t$. Let H^t be the set of all histories of length $t + 1$, $h^t = (s_0, a(s^0), s_1, a(s^1), \dots, s_{t-1}, a(s^{t-1}), s_t)$.

A (pure) *strategy* for player i is a mapping σ^i from game histories to actions feasible for agent i for the current state. That is, $\sigma^i(h^t) \in A^i(s_t)$. A *strategy profile* is a collection of strategies, one for each player.

I will denote by μ_σ the probability measure induced over game histories by π and a strategy profile σ . (Note that the restriction to pure strategies implies that the induced probability measure is made up entirely of mass points.) The single period payoff of agent i is

$$V^i(h^t|\sigma) := u \left(e^i(s_t) - \sum_{j \in \mathcal{I}} \sigma_j^i(h^t) + \sum_{j \in \mathcal{I}, j \neq i} \sigma_i^j(h^t) \right).$$

The continuation payoff is

$$\mathcal{V}^i(h^t|\sigma) := \sum_{\tau=t}^{\infty} \int_{h^\tau > h^t} \beta^{\tau-t} V^i(h^\tau|\sigma) \mu_\sigma(h^\tau|h^t),$$

where μ_σ denotes the probability measure induced over game histories by π and a strategy profile σ . Note that disposal of the good may be accomplished at h^t by agent i by setting $\sigma_i^i(h^t) > 0$.

A (pure strategy) *subgame perfect equilibrium (SPE)* is a strategy profile σ such that, for each i , h^t , and $\tilde{\sigma} := (\tilde{\sigma}^i, \sigma^{-i})$,

$$\mathcal{V}^i(h^t|\sigma) \geq \mathcal{V}^i(h^t|\tilde{\sigma}),$$

where $\tilde{\sigma}^i$ is any alternative strategy for agent i . Let $\Sigma(s)$ be the set of all SPEs starting from state s . Write $\Omega(s)$ for the set of all vectors $w \in \mathbb{R}^I$ such that there is a $\sigma \in \Sigma(s)$ that gives continuation payoff w^i to each player i .

16 Efficiency and Equilibrium under Arbitrary Punishment Conventions

Let ω be a stochastic process on \mathbb{R}^I . I will say that ω is a *feasible punishment convention* if, for each i and $s^t \geq s_0$, there is a $w \in \Omega(s_t)$ such that $w^i = \omega^i(s^t)$. I will investigate the equilibria that can be supported by reversion in the period following a deviation at s^{t-1} by i to some continuation equilibrium that gives s^t -continuation payoff $\omega^i(s^t)$ to i . Note that the continuation equilibria supporting the punishments of the various players need not be the same at s^t ; that is, for $j \neq i$, the j th element of the w described in the definition need not equal $\omega^j(s^t)$.

Let $\tilde{\omega}$ be a stochastic process on \mathbb{R}^I . Say that an allocation c is *supported by $\tilde{\omega}$ from s_0* if c is feasible and

$$\sum_{\tau=t}^{\infty} \sum_{s^\tau \geq s^t} \beta^{\tau-t} u(c^i(s^\tau)) \pi(s^\tau | s_t) \geq u(e^i(s_t)) + \beta \sum_{s'} \tilde{\omega}^i(s^t, s') \pi(s' | s_t)$$

for all i and $s^t \geq s_0$; in this case, write $c \in \mathcal{P}(s_0; \tilde{\omega})$.

The following lemma can be seen as a result of the one-deviation property of infinite games.

Lemma 15 *If c is supported by a feasible punishment convention ω from s_0 , then there is an SPE that implements c on the equilibrium path starting from initial state*

s_0 .

Let $\tilde{\omega}$ be a stochastic process on \mathbb{R}^I . I will say that c is *efficient with respect to* $\tilde{\omega}$ at s_0 if c is maximal in $\mathcal{P}(s_0; \tilde{\omega})$ for

$$\sum_i \gamma^i \sum_{t=0}^{\infty} \sum_{s^t \geq s_0} \beta^t u(c^i(s^t)) \pi(s^t | s_0)$$

for some γ in the I -dimensional unit simplex. Although there is nothing in the definition requiring that c is implemented by some SPE, the previous lemma implies that this is the case whenever ω is a feasible punishment convention. In this paper, I apply the term *efficient* generically to an allocation to mean that it is efficient with respect to some feasible punishment convention.

For a given allocation c , define

$$\bar{q}(s^{t+1}|c) := \max_j \left\{ \beta \frac{u'(c^j(s^{t+1}))}{u'(c^j(s^t))} \pi(s_{t+1}|s_t) \right\}$$

and

$$\bar{Q}(s^{t+1}|c) := \bar{q}(s^1|c) \bar{q}(s^2|c) \cdots \bar{q}(s^{t+1}|c).$$

I will say that c has *high implied interest rates* (Alvarez and Jermann) or $c \in HIR$

if

$$\sum_{t=0}^{\infty} \sum_{s^t} \bar{Q}(s^t|c) \left[\sum_i c^i(s^t) \right] \pi(s^t) < \infty.$$

The primary result of this paper is the following one.

Proposition 16 *Given a feasible allocation c , suppose that (i) $\sum_i c^i(s^t) = 1$ for all s^t ; (ii) $\mathcal{U}^i(c|s^t) \geq \mathcal{U}_{aut}^i(s_t)$ for all i and s^t ; (iii) $c \in HIR$; and (iv) whenever*

$$\frac{u'(c^i(s^t))}{u'(c^i(s^{t-1}))} < \max_j \left\{ \frac{u'(c^j(s^t))}{u'(c^j(s^{t-1}))} \right\} \quad (45)$$

holds for $s^t > s_0$, we have

$$\mathcal{U}^i(c|s^t) \leq u(e^i(s^t)) + \beta \sum_{s'} \left[\max_{w \in \Omega(s')} w^i \right] \pi(s'|s_t).$$

Then there exists a feasible punishment convention ω such that c is efficient with respect to ω from s_0 .

Before giving the proof, I present several auxiliary results useful in the proof of the main one.

Lemma 17 *(The Abreu property) Suppose that c is a feasible allocation. Then there is an SPE that implements c if and only if $\mathcal{U}^i(c|s^t) \geq \mathcal{U}_{aut}^i(s_t)$ for all i and s^t .*

Proof. Necessity is obvious from the definition of an SPE, and the fact that $\sigma_j^i(h^t) = 0$ for all j and h^t defines a strategy that gives at least $\mathcal{U}_{aut}^i(s_t)$ to i at each history h^t .

To show sufficiency, I construct a "simple" (Abreu) pure strategy profile σ that implements c on its path. I then show that σ is an equilibrium. I will write $h(s^t)$ for the game history prescribed by σ for exogenous history s^t ; and I let \bar{H} be the set of all other histories.²⁰

I begin by describing play following a deviation: let $\sigma(h^t) = 0$ for all $h^t \in \bar{H}$. That such play describes an equilibrium for the subgame following from h^t is obvious, since all $h^\tau \geq h^t$ are in \bar{H} , and any unilateral deviation can be seen to hurt the deviating player.

Define $\delta^i(s^t) := e^i(s_t) - c^i(s^t)$; define $\mathcal{K}(s^t) := \{k \in \mathcal{I} | \delta^k(s^t) > 0\}$, and $\bar{\mathcal{K}}(s^t) =$

²⁰It would be more precise to construct the path of σ and the function $h(\cdot)$ by recursions, and then define σ for histories off the path. This could be done as follows. First let $h(s_0) = s_0$ and then define $\sigma(h(s_0))$. Then successive values $h(s_0, s')$ and $\sigma(h(s_0, s'))$ can be assigned recursively by setting

$$h(s^{t-1}, s') = (h(s^{t-1}), \sigma(h(s^{t-1})), s')$$

for each s' , and then defining $\sigma(h(s^t))$. That such an algorithm is available for the strategy profile I describe is obvious.

$\mathcal{I} \setminus \mathcal{K}(s^t)$. Let

$$\begin{aligned} \Delta(s^t) &: = \sum_{k \in \mathcal{K}(s^t)} \delta^k(s^t) \\ &= 1 - \sum_i c^i(s^t) - \sum_{k \in \bar{\mathcal{K}}(s^t)} \delta^k(s^t). \end{aligned}$$

Now for $k \in \bar{\mathcal{K}}(s^t)$, set $\sigma_j^k(h(s^t)) = 0$ for each j . For $k \in \mathcal{K}(s^t)$, set

$$\sigma_j^k(h(s^t)) = \begin{cases} 0 & \text{if } j \in \mathcal{K}(s^t), j \neq k \\ \{([1 - \sum_i c^i(s^t)] / \Delta(s^t)) \delta^k(s^t)\} & \text{if } j = k \\ -[\delta^j(s^t) / \Delta(s^t)] \delta^k(s^t) & \text{if } j \in \bar{\mathcal{K}}(s^t). \end{cases}$$

I have now described $\sigma(h^t)$ for all histories on and off the path, and it is easy to verify that σ implements c on the prescribed path.

To see that σ constitutes an SPE for histories on the path, note that a deviation at any history $h(s^t)$ gets i at most

$$u(e^i(s^t)) + \beta \sum_{s'} \mathcal{U}_{aut}^i(s') = \mathcal{U}_{aut}^i(s_t);$$

since the constructed strategy gives at least this much to i at $h(s^t)$ anyway, the one-deviation property implies that there can be no profitable deviation from σ , Q.E.D.

■

Corollary 18 $\Omega(s)$ is convex for each s .

Proof. The set of allocations that can be supported by strategies constructed as in the lemma above can be seen to be convex. The convexity of $\Omega(s)$ is then easy to establish from the continuity and concavity of $\mathcal{U}(\cdot|s)$ in allocations, and the fact that strategies permit free-disposal of the good. ■

Lemma 19 Let ω be a stochastic process on \mathbb{R}^I satisfying $\omega^i(s^t) \geq \mathcal{U}_{aut}^i(s_t)$ for all i and s^t . Suppose that there is a feasible allocation c such that $\mathcal{U}^i(c|s^t) \geq \omega^i(s^t)$ for all i and s^t . Then ω is a feasible punishment convention.

Proof. Fix s^t and i . Let c_{aut} be the autarkic allocation, and define $c_\lambda := \lambda c + (1 - \lambda) c_{aut}$ for $\lambda \in [0, 1]$. By the properties of $u(\cdot)$, it can be seen that the function $\lambda \mapsto \mathcal{U}^i(c_\lambda)$ is continuous and concave on $[0, 1]$.²¹ Therefore, there is a λ such that $\mathcal{U}^i(c_\lambda|s^t) = \omega^i(s^t)$. Moreover, it must be true that $\mathcal{U}^j(c_\lambda|s^t) \geq \mathcal{U}_{aut}^j(s_t)$ for all j and s^t . Therefore Lemma 17 shows that there is an SPE that implements $c_\lambda|s^t$; i.e., there is an SPE continuation that gives $\omega^i(s^t)$ to player i at s^t . Repeating this analysis for each i and s^t , proves the result. ■

Lemma 20 If c is a feasible allocation, and $\mathcal{U}^i(c|s^t) \geq \mathcal{U}_{aut}^i(s_t)$ for all s^t , then $u(c^i(s^t))$ is bounded.

²¹This is easily established after noting that c_{aut} is bounded away from zero.

Proof. Clearly, $u(c^i(s^t)) \leq u(1)$, so $\mathcal{U}^i(c|s^t) \leq u(1)/(1-\beta)$. Thus $\mathcal{U}^i(c|s^t) \geq \mathcal{U}_{aut}^i(s_t)$ implies that

$$\begin{aligned} u(c^i(s^t)) &\geq u(c^i(s_t)) + \beta \sum_{s'} [\mathcal{U}_{aut}^i(s') - \mathcal{U}^i(c|s^t, s')] \pi(s'|s_t) \\ &\geq u(c^i(s_t)) + \beta \sum_{s'} [\mathcal{U}_{aut}^i(s') - u(1)/(1-\beta)] \pi(s'|s_t) \\ &> -\infty. \end{aligned}$$

The result follows from the fact that \mathcal{S} is finite. ■

Proof of the Proposition. The proof is constructive.

I begin by setting $\omega^i(s_0, s') := \mathcal{U}_{aut}^i(s')$ for each i .²² Other values of $\omega(s^t)$ are set according to the following algorithm.

Fix i and $s^t > s_0$. If

$$\frac{u'(c^i(s^t))}{u'(c^i(s^{t-1}))} = \max_j \left\{ \frac{u'(c^j(s^t))}{u'(c^j(s^{t-1}))} \right\}, \quad (46)$$

set $\omega^i(s^t, s') = \mathcal{U}_{aut}^i(s')$ for each s' . If instead (45) holds, then by the hypothesis of

²²The punishment convention will be constructed so that the enforcement constraints do not bind at s_0 .

the proposition,

$$\begin{aligned}
\mathcal{U}_{aut}^i(s_t) & : = u(e^i(s^t)) + \beta \sum_{s'} \mathcal{U}_{aut}^i(s') \pi(s'|s_t) \\
& \leq \mathcal{U}^i(c|s^t) \\
& \leq u(e^i(s^t)) + \beta \sum_{s'} \left[\max_{w \in \Omega(s')} w^i \right] \pi(s'|s_t).
\end{aligned}$$

By the convexity of $\Omega(s')$ (Corollary 18) and the fact that autarky can be supported as an equilibrium (Lemma 17), there exists a selection $w : \mathcal{S} \rightarrow \mathbb{R}^I$ from Ω (i.e., a function with $w(s) \in \Omega(s)$ for each s) such that

$$\mathcal{U}^i(c|s^t) = u(e^i(s^t)) + \beta \sum_{s'} w^i(s') \pi(s'|s_t),$$

(Note that Lemma 17 implies that $w^j(s') \geq \mathcal{U}_{aut}^j(s')$ for each j and s' , because $w(s') \in \Omega(s')$.) Choose some such function and set $\omega^i(s^t, s') = w^i(s')$ for each s' .

Repeating the procedure for each i and $s^t > s_0$ such that (45) holds defines a function ω . It follows from Lemma 19 that ω defines a feasible punishment convention.

An allocation that is efficient with respect to ω solves a programming problem of the form

$$\max_i \sum_i \gamma^i \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u(c^i(s^t)) \pi(s^t|s_0)$$

subject to

$$\sum_i c^i(s^t) \leq 1$$

for all s^t , and

$$\mathcal{U}^i(c|s^t) \geq u(e^i(s^t)) + \beta \sum_{s'} \omega^i(s^t, s') \pi(s'|s_t)$$

for all i and s^t . The Lagrangian function for this problem can be written as

$$\begin{aligned} \mathcal{L}(c, \lambda, \eta) \quad : \quad &= \sum_i \gamma^i \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u(c^i(s^t)) \pi(s^t|s_0) \\ &+ \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \lambda(s^t) \left[1 - \sum_i c^i(s^t) \right] \pi(s^t|s_0) \\ &+ \sum_i \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \eta^i(s^t) \left\{ \sum_{\tau=t}^{\infty} \sum_{s^\tau} \beta^{\tau-t} u(c^i(s^\tau)) \pi(s^\tau|s_t) \right. \\ &\quad \left. - \left[u(e^i(s^t)) + \beta \sum_{s'} \omega^i(s^t, s') \pi(s'|s_t) \right] \pi(s^t|s_0) \right\}. \end{aligned}$$

To show that c solves such a programming problem it suffices (by Theorem 2 on page 221 of Luenberger) to find γ and multipliers (λ, η) such that (c, λ, η) constitutes a saddle point of $\mathcal{L}(c, \lambda, \eta)$.²³ I begin by defining appropriate weights and multipliers.

²³Note that, for the purpose of the Theorem of Luenberger, the Lagrange multipliers are the sequences whose elements are $\beta^t \lambda(s^t) \pi(s^t|s_0)$ and $\beta^t \eta^i(s^t) \pi(s^t|s_0)$. It follows from (iii) that each of these sequences constructed in the proof is summable, so that each sequence defines an element of the norm dual space of l_∞ .

Define $\gamma \in \Delta^I$ by

$$\gamma^i u' (c^i (s_0)) = \gamma^j u' (c^j (s_0))$$

for all i and j . Let $\eta^i (s_0) = 0$, and define $\eta^i (s^t, s')$ recursively according to

$$\frac{[\gamma^i + \sum_{\tau=0}^t \eta^i (s^\tau) + \eta^i (s^t, s')] u' (c^i (s^t, s'))}{[\gamma^i + \sum_{\tau=0}^t \eta^i (s^\tau)] u' (c^i (s^t))} = \max_j \frac{u' (c^j (s^t, s'))}{u' (c^j (s^t))}$$

for each i . Note that $\eta \geq 0$, and that $\eta^i (s^{t+1}) = 0$ whenever (46) holds, or whenever

$$\mathcal{U}^i (c|s^t) > u (c^i (s^t)) + \beta \sum_{s'} \omega^i (s^t, s') \pi (s'|s_t).$$

Finally, define

$$\lambda (s^t) = \left[\gamma^i + \sum_{\tau=0}^t \eta^i (s^\tau) \right] u' (c^i (s^t));$$

note that the expression on the right-hand side is independent of i .

By construction, the multipliers (λ, η) can be seen to minimize $\mathcal{L} (c, \cdot, \cdot)$ over all non-negative alternatives. It remains to verify that c maximizes $\mathcal{L} (\cdot, \lambda, \eta)$.

From Lemma 20, $|u (c^i (s^t))|$ is bounded. It follows that the sum

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \eta^i (s^t) \left\{ \sum_{\tau=t}^{\infty} \sum_{s^\tau} \beta^{\tau-t} u (c^i (s^\tau)) \pi (s^\tau | s_t) \right\}$$

converges absolutely, since

$$\left| \beta^t \eta^i(s^t) \left\{ \sum_{\tau=t}^{\infty} \sum_{s^\tau} \beta^{\tau-t} u(c^i(s^\tau)) \pi(s^\tau | s_t) \right\} \right| \\ \leq \beta^t \eta^i(s^t) \left\{ \sum_{\tau=t}^{\infty} \sum_{s^\tau} \beta^{\tau-t} |u(c^i(s^\tau))| \pi(s^\tau | s_t) \right\},$$

and the Right-hand side is bounded. Thus (e.g., by Theorem 3.55 of Rudin (p.78))

terms in the expression may be rearranged without changing the value of the sum.

Now showing that c maximizes $\mathcal{L}(\cdot, \lambda, \eta)$ may be seen as equivalent to showing that

$$\sum_i \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left\{ \left[\gamma^i + \sum_{\tau=0}^t \eta^i(s^\tau) \right] u(c^i(s^t)) - \lambda(s^t) c^i(s^t) \right\} \\ - \sum_i \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left\{ \left[\gamma^i + \sum_{\tau=0}^t \eta^i(s^\tau) \right] u(\tilde{c}^i(s^t)) - \lambda(s^t) \tilde{c}^i(s^t) \right\}$$

is non-negative for all allocations \tilde{c} . Now using the definition of $\lambda(s^t)$, and combining

and rearranging the terms, this expression is seen to equal

$$\sum_i \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \\ \times \left\{ \left[\gamma^i + \sum_{\tau=0}^t \eta^i(s^\tau) \right] \{ u(c^i(s^t)) - u'(\tilde{c}^i(s^t)) [\tilde{c}^i(s^t) - c^i(s^t)] - u(\tilde{c}^i(s^t)) \} \right\}$$

By the concavity of u , this expression is non-negative, Q.E.D. ■

17 Decentralization

The notion of market equilibrium introduced in this section is a generalization of that studied by Alvarez and Jermann.

A portfolio of contingent claims for agent i is a stochastic process b^i with $b^i(s^t) \in \mathbb{R}$. I write b for the profile of agents' portfolios.

An (Arrow) *price system* is a positive stochastic process q , where $q(s^t)$ is interpreted as the price after (exogenous) history s^{t-1} of a claim to a unit of the good after history s^t .

A *system of solvency constraints* is a stochastic process d with $d(s^t) \in \mathbb{R}^I$.

A (*market*) *equilibrium with solvency constraints* is a consumption allocation c , a profile of portfolios b , a price system q , and a system of solvency constraints d such that

1. for each s^t and i , $(c^i(s^t), b^i(s^t, \cdot))$ solves

$$J_t^i(b^i(s^t), s^t) = \max u(\tilde{c}^i) + \beta \sum_{s'} J_{t+1}^i(\tilde{b}^i(s'), (s^t, s')) \pi(s'|s_t)$$

subject to

$$\tilde{c}^i + \sum_{s'} q(s^t, s') \tilde{b}^i(s') \leq e^i(s_t) + b^i(s^t)$$

and

$$\tilde{b}^i(s') \geq d^i(s^t, s')$$

for each s' ; and

2. the goods and assets markets clear for each s^t ,

$$\sum_i c^i(s^t) = 1$$

and

$$\sum_i b^i(s^t) = 0.$$

Let $\tilde{\omega}$ be an I -vector stochastic process; that is, $\tilde{\omega}(s^t) \in \mathbb{R}^I$. I will say that the solvency constraints d are *appropriate for punishment convention* $\tilde{\omega}$ if

$$J_t^i(d^i(s^t), s^t) = u(e^i(s_t)) + \beta \sum_{s'} \tilde{\omega}^i(s^t, s') \pi(s'|s_t)$$

for all i, t , and s^t

An important result of Alvarez and Jermann is a version of the Second Welfare Theorem. They show that an allocation that is efficient with respect to the

punishment convention defined by autarkic consumption, that is,

$$\omega_{aut}^i(s^t) := \mathcal{U}_{aut}^i(s_t),$$

and that has high implied interest rates can be decentralized by an equilibrium with solvency constraints that are appropriate for ω_{aut} . The following proposition generalizes this result for an environment with an arbitrary punishment convention.

Proposition 21 *Suppose that c is efficient with respect to a feasible punishment ω , and that $c \in HIR$. Then there exists a profile of portfolios b , a price system q , and a system of solvency constraints d such that (c, b, q, d) constitute an equilibrium with solvency constraints that are appropriate for ω .*

Proof. The proof is identical to that of Proposition 4.1 of Alvarez and Jermann (2000). ■

18 Discussion

The fourth condition of the hypothesis of Proposition 16 merits some discussion. This condition is just the requirement that, whenever the rationalization of c requires that the enforcement constraint be binding, there is *some* equilibrium path that begins with $e^i(s^t)$ that gives a payoff at least as good as $c|s^t$. One implication is

that, in a period in which his enforcement constraint binds, i 's consumption cannot be too much *bigger* than his endowment income; specifically, c must satisfy

$$u'(c^i(s^t)) - u'(e^i(s^t)) \leq \beta \sum_{s'} \left\{ \max_{w \in \Omega(s')} w^i - \mathcal{U}^i(c|(s^t, s')) \right\} \pi(s'|s_t).$$

(Note that (i) and (ii) imply that c is an equilibrium path, so that the term on the Right-hand side is non-negative.) Violation of this condition means that feasible punishments are too harsh to explain why i is constrained at s^t .

19 Conclusion

The main result of the paper identifies a set of conditions under which it is possible to rationalize a given consumption allocation as an efficient outcome of a simple multilateral exchange game subject to an exogenously specified convention for punishing misbehavior. As in Kocherlakota (1996), I assume that society chooses an allocation efficiently (in some sense) from among those implemented in a subgame perfect equilibrium. As a novelty, I consider the consequences of inefficiencies off the equilibrium path.

A heuristic motivation for studying such an environment is the notion that, while markets may facilitate the arrival of the economy at a locally efficient outcome, there

may be a less effective coordination device selecting continuations to be followed off the equilibrium path. Since these "punishment" continuations determine which paths may be sustained in equilibrium, this feature is obviously important.

I believe that more thorough study of the form of punishments for misbehavior is important for two reasons. First, it is likely that economic agents view the consequences of default in a more complex and interesting way than can be captured by "autarkic consumption". Second, Proposition 16 informs us that there is a lot of latitude in what may be explained by feasible punishments, and thus potentially a lot to be gained by being careful about modeling them.

My own research agenda includes study of calibrated economies with more carefully modeled alternatives to market participation. I also find compelling the possibility of the recovery from data of punishments rationalizing an observed pattern of consumption within a cohort. Working backwards from data will help us to understand the value of the model, and the nature of enforcement institutions.

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Vita

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